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Qquasi-convolution of certain multivalent functions with a fixed point

Asraa Abdul Jaleel Husien
Technical Institute of Al_ Diwaniy
Al_Furat Al_ Awsat Technical University
Diwaniya
Iraq

Abstract

In this research, we reach new results correlated with quasi convolution of certain multivalent functions with a fixed point.

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1. Introduction

Let $T_\rho^*(w)$ denote the class of functions of the form:

$$\mathcal{F}(z) = a_\rho(z-w)^\rho - \sum_{n=1}^{\infty} a_{n+\rho}\ (z-w)^{n+\rho}\ (a_\rho > 0;\ a_{n+\rho} \geq 0;\ \rho \in N), \tag{1}$$

$$\mathcal{F}_{\mathrm{i}}(z) = a_{\rho,\mathrm{i}}(z-w)^\rho - \sum_{n=1}^{\infty} a_{n+\rho,\mathrm{i}}\ (z-w)^{n+\rho}\ (a_{\rho,\mathrm{i}} > 0;\ a_{n+\rho,\mathrm{i}} \geq 0;\ \rho \in N), \tag{2}$$

$$G(z) = b_\rho(z-w)^\rho - \sum_{n=1}^{\infty} b_{n+\rho}\ (z-w)^{n+\rho}\quad (b_\rho > 0;\ b_{n+\rho} \geq 0;\ \rho \in N), \tag{3}$$

and

$$G_{\mathrm{j}}(z) = b_{\rho,d}(z-w)^\rho - \sum_{n=1}^{\infty} b_{n+\rho,\mathrm{j}}\ (z-w)^{n+\rho}\ (b_{\rho,\mathrm{j}} > 0;\ b_{n+\rho,\mathrm{j}} \geq 0;\ \rho \in N). \tag{4}$$

E-mail: asraalsade2@gmail.com, esraabdul@atu.edu.iq

which are analytic in the open unit disc $\mathbb{U} = \{z : \backslash z \backslash < 1\}$ where w is a fixed point in $\mathbb{U}$.

In [3] Kanas and Ronning introduced classes $\mathcal{S}_\rho^*(w)$ and $\mathcal{C}_\rho^*(w)$ starlike and convex functions using the normalization $f(w) = f'(w) - 1 = 0$, where $w \in \mathbb{U}$ is a fixed point and if $(\xi = 0,\ \rho = 1)$, the following classes of functions of w–starlike and w–convex respectively

$$\mathcal{S}_\rho^*(w) = \left\{ \mathcal{F}(z) \in T_\rho^*(w) :\ \mathrm{Re}\left\{ \frac{(z-w)\mathcal{F}'(z)}{\mathcal{F}(z)} \right\} > \xi,\ \ 0 \le \xi < \rho\ ,\ \ z \in \mathbb{U} \right\},$$

$$\mathcal{C}_\rho^*(w) = \left\{ \mathcal{F}(z) \in T_\rho^*(w) : \mathrm{Re}\left\{ 1 + \frac{(z-w)\mathcal{F}''(z)}{\mathcal{F}'(z)} \right\} > \xi,\ \ 0 \le \xi < \rho,\ z \in \mathbb{U} \right\}.$$

And w is a fixed point in $\mathbb{U}$. Also were introduced Malih and Abed [4] and Panwar and Reena [5] Various studies on the fixed point.

Now, we define $\mathcal{S}_\rho^*(A, B, w)$ be the subclass of $\mathcal{S}_\rho^*(w)$ for the functions $\mathcal{F}(z)$ in (1), which satisfy:

$$\left| \frac{(\mathrm{z-w})\acute{\mathcal{F}}(\mathrm{z}) - \rho\mathcal{F}(\mathrm{z})}{A\rho\mathcal{F}(z) + B(z-w)\acute{\mathcal{F}}(z)} \right| < \ell, \quad (z \in \mathbb{U},\ -1 \le A < B \le 1,\ 0 < \ell \le 1 \text{ and } \rho \in N),$$

and a functions $\frac{(z-w)\acute{\mathcal{F}}(z)}{\rho} \in \mathcal{S}_\rho^*(A, B, w)$ be in the class $\mathcal{C}_\rho^*(A, B, w)$. Now :

$\mathcal{F}(z) \in \mathcal{S}_\rho^*(A, B, w)$ $(-1 \le A < B \le 1, 0 < \ell \le 1 \text{ and } \rho \in N)$ if and only if

$$\sum_{n=1}^{\infty} [\ell(A\rho + B(n+\rho)) + n] a_{n+\rho} \le \ell\rho(A+B) a_\rho. \tag{5}$$

Easy to prove that $\mathcal{F}(z) \in \mathcal{C}_\rho^*(A, B, w)$ $(-1 \le A < B \le 1,\ 0 < \ell \le 1 \text{ and } \rho \in N)$ if and only if

$$\sum_{n=1}^{\infty} \left[1 + \frac{n}{\rho} \right] [\ell(A\rho + B(n+\rho)) + n] a_{n+\rho} \le \ell\rho(A+B) a_\rho. \tag{6}$$

And the functions $\mathcal{F}(z)$ in (1) be in the class $\mathcal{S}_{\rho,r}^*(A, B, w)$ if and only if

$$\sum_{n=1}^{\infty} \left[1 + \frac{n}{\rho} \right]^r [\ell(A\rho + B(n+\rho)) + n] a_{n+\rho} \le \ell\rho(A+B) a_\rho. \tag{7}$$

where r is an nonnegative real number.

We note that for every nonnegative real number r, a class $\mathcal{S}^{*}_{\rho,r}(A,B,w)$ is nonempty as the functions of the form

$$\mathcal{F}(z) = a_{\rho}(z-w)^{\rho} - \sum_{n=1}^{\infty} \frac{\ell\rho(A+B)a_{\rho}}{\left[1+\frac{n}{\rho}\right]^{r}[\ell(A\rho+B(n+\rho))+n]} \lambda_{n}(z-w)^{n+\rho},$$

$$\left(a_{\rho} > 0;\ \lambda_{n} \geq 0 \text{ and } \sum_{n=1}^{\infty}\lambda_{n} \leq 1\right). \tag{8}$$

Satisfy the inequality (7). We get that

1. $\mathcal{S}^{*}_{\rho,1}(A,B,w) \equiv \mathcal{C}^{*}_{\rho}(A,B,w)$ and $\mathcal{S}^{*}_{\rho,0}(A,B,w) \equiv \mathcal{S}^{*}_{\rho}(A,B,w)$,
2. $\mathcal{S}^{*}_{\rho,r_1}(A,B,w) \subset \mathcal{S}^{*}_{\rho,r_2}(A,B,w)$ $(r_{1>}r_2 \geq 0)$,
3. $\mathcal{S}^{*}_{\rho,r}(A,B,w) \subset \mathcal{S}^{*}_{\rho,r-1}(A,B,w) \subset \ldots \subset \mathcal{S}^{*}_{\rho}(A,B,w) \subset \mathcal{C}^{*}_{\rho}(A,B,w)$.

As the Hadamard product (or convolution) of $\mathcal{F}(z)$ and $G(z)$ is given by

$$(\mathcal{F}*G)(z) = (G*\mathcal{F})(z) = a_{\rho}b_{\rho}(z-w)^{\rho} - \sum_{n=1}^{\infty} a_{n+\rho}b_{n+\rho}\ (z-w)^{n+\rho}. \tag{9}$$

So we know a quasi- convolution for multiple functions

$$(\mathcal{F}_1 * \mathcal{F}_2 * \ldots * \mathcal{F}_t * G_1 * G_2 * \ldots * G_d)(z) = K(z) =$$
$$\prod_{\mathrm{i}=1}^{t} a_{\rho,\mathrm{i}} \prod_{\mathrm{j}=1}^{d} b_{\rho,\mathrm{j}}\ (z-w)^{\rho} - \sum_{n=1}^{\infty}\prod_{\mathrm{i}=1}^{t} a_{n+\rho,\mathrm{i}} \prod_{\mathrm{j}=1}^{d} b_{n+\rho,\mathrm{j}}\ (z-w)^{n+\rho}. \tag{10}$$

note that w = 0, the function $\mathcal{F}(z)$ have been introduced by [2], we note other studies of various other classes with different results in [1, 6, 7].

2. Main Results

Note that in all the following results that $-1 \leq A < B \leq 1$, $z \in \mathbb{U}$, r is any fixed nonnegative real number, and w is a fixed point in $\mathbb{U}$.

Theorem 1 : *Letting a functions* $\mathcal{F}_{\mathrm{i}}(z)$ *in (2) such that* $F_i(z) \in S^{*}_{(\rho,r)}(A,B,w)$ $(\forall i = 1, 2, n, t)$ *and Letting a functions* $G_j(z)$ *in (4) such that* $G_{\mathrm{j}}(z) \in \mathcal{C}^{*}_{\rho}(A,B,w)$ $(\forall\, \mathrm{j} = 1,2,\ldots,d)$, *then the quasi– convolution* $K(z) \in \mathcal{S}^{*}_{\rho,(r+1)t+2d-1}(A,B,w)$.

Proof : By K(z) in (10), must we proved

$$\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{(r+1)t+2d-1}[\ell(A\rho+B(n+\rho))+n]\left[\prod_{\mathrm{i}=1}^{t}a_{n+\rho,\mathrm{i}}\prod_{\mathrm{j}=1}^{d}b_{n+\rho,\mathrm{j}}\right]\right\}$$
$$\leq \ell\rho(A+B)\left[\prod_{\mathrm{i}=1}^{t}a_{\rho,\mathrm{i}}\prod_{\mathrm{j}=1}^{d}b_{\rho,\mathrm{j}}\right]. \quad (1)$$

$\mathcal{F}_{\mathrm{i}}(z)\in\mathcal{S}^{*}_{\rho,r}(A,B,w)$, and by (7), then

$$\sum_{n=1}^{\infty}\left[1+\frac{n}{\rho}\right]^{r}[\ell(A\rho+B(n+\rho))+n]a_{n+\rho,\mathrm{i}}\leq \ell\rho(A+B)a_{\rho,\mathrm{i}}, \quad (12)$$

$$\left[1+\frac{n}{\rho}\right][\ell(A\rho+B(n+\rho))+n]a_{n+\rho,\mathrm{i}}\leq \ell\rho(A+B)a_{\rho,\mathrm{i}}, \quad (13)$$

or

$$a_{n+\rho,\mathrm{i}}\leq\frac{\ell\rho(A+B)}{\left[1+\frac{n}{\rho}\right][\ell(A\rho+B(n+\rho))+n]}a_{\rho,\mathrm{i}}, \quad (14)$$

$(\forall\,\mathrm{i}=1,2,\dots,t)$. The right hand side expression of this last inequality is not greater that $\left[1+\frac{n}{\rho}\right]^{-(r+1)}$. Then

$$a_{n+\rho,\mathrm{i}}\leq\left[1+\frac{n}{\rho}\right]^{-(r+1)}a_{\rho,\mathrm{i}}, \qquad (\forall\,\mathrm{i}=1,2,\dots,t). \quad (15)$$

$G_{\mathrm{j}}(z)\in\mathcal{C}^{*}_{\rho}(A,B,w)$ and by (6), then

$$\sum_{n=1}^{\infty}\left[1+\frac{n}{\rho}\right][\ell(A\rho+B(n+\rho))+n]b_{n+\rho,\mathrm{j}}\leq \ell\rho(A+B)b_{\rho,\mathrm{j}}. \quad (16)$$

$(\forall\,\mathrm{j}=1,2,\dots,d)$. we get

$$b_{n+\rho,\mathrm{i}}\leq\left[1+\frac{n}{\rho}\right]^{-2}b_{\rho,\mathrm{i}}. \quad (17)$$

Letting (15) $(\forall\,\mathrm{i}=1,2,\dots,t-1)$ and $\mathrm{i}=t$, (17) $(\forall\mathrm{j}=1,2,\dots,d)$, we have

$$\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{(r+1)t+2d-1}\left[\ell(A\rho+B(n+\rho))+n\right]\left[\prod_{i=1}^{t}a_{n+\rho,i}\ \prod_{j=1}^{d}b_{n+\rho,j}\right]\right\}$$

$$\leq\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{(r+1)t+2d-1}\left[\ell(A\rho+B(n+\rho))+n\right]a_{n+\rho,t}\right.$$

$$\left.\times\left(\left[1+\frac{n}{\rho}\right]^{-(r+1)(t-1)}\left[1+\frac{n}{\rho}\right]^{-2d}\prod_{i=1}^{t-1}a_{\rho,i}\ \prod_{j=1}^{d}b_{\rho,j}\right)\right\}$$

$$=\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{r}[\ell(A\rho+B(n+\rho))+n]a_{n+\rho,t}\prod_{i=1}^{t-1}a_{\rho,i}\ \prod_{j=1}^{d}b_{\rho,j}\right\}$$

$$\leq\ell\rho(A+B)\quad\left[\prod_{i=1}^{t}a_{\rho,i}\prod_{j=1}^{d}b_{\rho,j}\right].\tag{18}$$

Then, we obtain $K(z)\in\mathcal{S}^{*}_{\rho,(r+1)t+2d-1}(A,B,w)$.

We put $r=0$ in theorem 1, we have

Corollary 2 : *Letting a functions* $\mathcal{F}_i(z)$ *in (2) such that* $\mathcal{F}_i(z)\in\mathcal{S}^{*}_{\rho}(A,B,w)$ $(\forall\, i=1,2,\dots,t)$, *and Letting a functions* $G_j(z)$ *in (4) such that* $G_j(z)\in\mathcal{C}^{*}_{\rho}(A,B,w)$ $(\forall\, j=1,2,\dots,d)$. *then the quasi– convolution* $K(z)\in\mathcal{S}^{*}_{\rho,t+2d-1}(A,B,w)$.

Corollary 3 : *Letting a functions* $\mathcal{F}_i(z)$ *in (2) such that* $\mathcal{F}_i(z)\in\mathcal{S}^{*}_{\rho}(A,B,w)$ $(\forall\, i=1,2,\dots,t)$, *then the quasi–convolution* $(\mathcal{F}_1*\mathcal{F}_2*\dots*\mathcal{F}_t)(z)\in\mathcal{S}^{*}_{\rho,t-1}(A,B,w)$.

Corollary 4 : *Letting a functions* $G_j(z)$ *in (4) such that* $G_j(z)\in\mathcal{C}^{*}_{\rho}(A,B,w)$ $(\forall\, j=1,2,\dots,d)$, *then the quasi– convolution* $(G_1*G_2*\dots*G_d)\ (z)\in\mathcal{S}^{*}_{\rho,2d-1}$ (A,B,w).

Theorem 5 : *Letting a functions* $\mathcal{F}_i(z)$ *in (2)such that* $\mathcal{F}_i(z)\in\mathcal{S}^{*}_{\rho}(A,B,w)$ $(\forall\, i=1,2,\dots,t)$, *and Letting a functions* $G_j(z)$ *in (4)such that* $G_j(z)\in\mathcal{S}^{*}_{\rho,r}$ (A,B,w) $(\forall\, j=1,2,\dots,d)$. *then the quasi– convolution* $K(z)\in\mathcal{S}^{*}_{\rho,t+(r+1)d-1}$ (A,B,w).

Proof : by K(z) in (10), must we proved

$$\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{t+(r+1)d-1}[\ell(A\rho+B(n+\rho))+n]\left[\prod_{i=1}^{t}a_{n+\rho,i}\ \prod_{j=1}^{d}b_{n+\rho,j}\right]\right\}$$

$$\leq \ell\rho(A+B)\left[\prod_{i=1}^{t} a_{\rho,i}\prod_{j=1}^{d} b_{\rho,j}\right]. \tag{19}$$

As $\mathcal{F}_i(z)\in\mathcal{S}_\rho^*(A,B,w)$, and by (5), then

$$\sum_{n=1}^{\infty}[\ell(A\rho+B(n+\rho))+n]a_{n+\rho,i}\leq\ell\rho(A+B)a_{\rho,i}\ , \tag{20}$$

$(\forall\, i=1,2,\ldots,t)$. we get

$$a_{n+\rho,i}\leq\left[1+\frac{n}{\rho}\right]^{-1}a_{\rho,i}\,,\qquad (\forall\, i=1,2,\ldots,t). \tag{21}$$

And as $G_j(z)\in\mathcal{S}_{\rho,r}^*(A,B,w)$ and by (7), then

$$\sum_{n=1}^{\infty}\left[1+\frac{n}{\rho}\right]^{r}[\ell(A\rho+B(n+\rho))+n]b_{n+\rho,j}\leq\ell\rho(A+B)b_{\rho,j}. \tag{22}$$

$(\forall j=1,2,\ldots,d)$. we have

$$b_{n+\rho,j}\leq\left[1+\frac{n}{\rho}\right]^{-(r+1)}b_{\rho,j},\qquad (\forall j=1,2,\ldots,d). \tag{23}$$

Letting (21) $(\forall\, i=1,2,\ldots,t)$, (23) $(\forall j=1,2,\ldots,d-1)$ and $j=d$, we have

$$\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{t+(r+1)d-1}[\ell(A\rho+B(n+\rho))+n]\left[\prod_{i=1}^{t}a_{n+\rho,i}\ \prod_{j=1}^{d}b_{n+\rho,j}\right]\right\}$$

$$\leq\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{t+(r+1)d-1}[\ell(A\rho+B(n+\rho))+n]b_{n+\rho,d}\right.$$

$$\left.\times\left(\left[1+\frac{n}{\rho}\right]^{-t}\left[1+\frac{n}{\rho}\right]^{-(r+1)(d-1)}\prod_{i=1}^{t-1}a_{\rho,i}\ \prod_{j=1}^{d}b_{\rho,j}\right)\right\}$$

$$=\sum_{n=1}^{\infty}\left\{\left[1+\frac{n}{\rho}\right]^{r}[\ell(A\rho+B(n+\rho))+n]b_{n+\rho,d}\prod_{i=1}^{t}a_{\rho,i}\ \prod_{j=1}^{d-1}b_{\rho,j}\right\}$$

$$\leq\ell\rho(A+B)\left[\prod_{i=1}^{t}a_{\rho,i}\prod_{j=1}^{d}b_{\rho,j}\right]. \tag{24}$$

Then, we obtain $K(z)\in\mathcal{S}^*_{\rho,t+(r+1)d-1}(A,B,w)$.
We put $r=0$ in theorem 5, we have

Corollary 6 : *Letting a functions* $\mathcal{F}_i(z)$ *in (2), and a functions* $G_j(z)$ *in (4)such that* $\mathcal{F}_i(z), G_j(z)$ *belong to* $\mathcal{S}^*_\rho(A,B,w)$ $(\forall\, i = 1,2,\ldots,t;\ \forall\, j = 1,2,\ldots,d)$. *then the quasi– convolution* $K(z) \in \mathcal{S}^*_{\rho,t+d-1}(A,B,w)$.

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