THE CORRELATION BETWEEN THE ROTOR SHAPE AND THE ENERGETIC PERFORMANCE OF A ROTATING MACHINE WITH PROFILED ROTORS

Prof. Nicolae Băran, PhD Eng.
Affiliation: Politehnica University of Bucharest
Post address: str. Splaiul Independenței nr. 313, sector 6, Bucharest
E-mail:n_baran_fimm@yahoo.com
Antonios Detzortzis, PhD Student Eng.
Affiliation: Politehnica University of Bucharest
Post address: str. Splaiul Independenței nr. 313, sector 6, Bucharest
E-mail:tony@elrom.net
Malik N. Hawas, PhD Student Eng.
Affiliation: Foundation of Technical Education, Iraq
Post address: str. Splaiul Independenței nr. 313, sector 6, Bucharest
E-mail:maliknhawas@yahoo.com
Ș.L. Daniel Besnea, PhD Student Eng.
Affiliation: Politehnica University of Bucharest
Post address: str. Splaiul Independenței nr. 313, sector 6, Bucharest
E-mail:d_bes@yahoo.com

Abstract - In this paper, for a rotating machine with two profiled rotors, the influence of the rotor shape on the volumetric flow rate and on the driving power of the machine is showed. If the radius and the rotor length are considered constant, the rotating piston shape influences the flow rate circulated by the machine. The rotor shape in two versions is presented; for each constructive version the power lost by viscous friction between the rotors and the case is calculated.

Keywords: Rotating machine, profiled rotors, energetic performance, volumetric flow rate, driving power.

1. Introduction

The machines are units used for the energy conversion from one form into another by means of a mobile body (piston, profiled rotor, and paddle). By the purpose, the machines are divided into two categories [1] [2] [3]:

1. Force machine (motor machine), which converts a particular form of mechanical energy.

2. Working machine that converts mechanical energy into another form of energy.

By the flow parameters variation, working and force machines that are circulated by fluids are classified as follows:

a) Hydraulic machine which neglects the thermal phenomena;

b) Thermal machines which neglects the occurring thermal phenomena;

By the operating principle, thermal machines are divided into three categories:

a) Reciprocation piston machine;

b) Rotating machines (with profiled rotors or blades);

c) Reaction machines.

This paper presents a working machine, namely a thermal machine with profiled rotating rotors; the “machine” term, refers to the fact that it can be used as a pump, fan or low pressure compressor.

Currently, the researchers aim to implement the rotating machines in technics because it has the following advantages:

- It are simple in terms of construction; (no valves, the crank drive mechanism is eliminated);

- The torque received at the working machine shaft (M) from the outside is almost entirely used for fluid pressure energy increasing because $\vec{M} = \vec{F} \times \vec{b}$; $M = F \cdot b \cdot sin\alpha$. The force (F) exerced by the piston is always pependicular (α=90oC) on the force arm (b).

The machine energetic performances (flow rate, power, and efficiency) are influenced by two types of parameters:

- the geometric parameters: rotor length, rotating piston height, and the rotor radius;

-the functional parameters: the machine rpm, the increase in fluid pressure between the machine suction and discharge.

In this paper will follow only two aspects:

-The influence of rotating piston shape on the flow rate circulated by the machine.

-The influence of the rotor shape on the machine driving the power.

2. The machine sketch and the operating principle

In Figure 1 the sketch of the rotating machine with profiled rotors is observed. The rotors, after their rotation by an angle at the centre equal to 90° are drawn in Figure 1 in successive positions (a, b, c).

In order to determine the volumetric flow rate of the machine circulated gas, in Figure 1 it is observed that two volumes will be transported from the suction to the discharge at one shaft rotation:

$$V_u = \left(\pi \cdot r_c^2 - \pi \cdot r_r^2\right) \cdot l \left[\mathrm{m}^3 / \mathrm{rot}\right] \qquad (1)$$

replacing rc = rr + z it results:

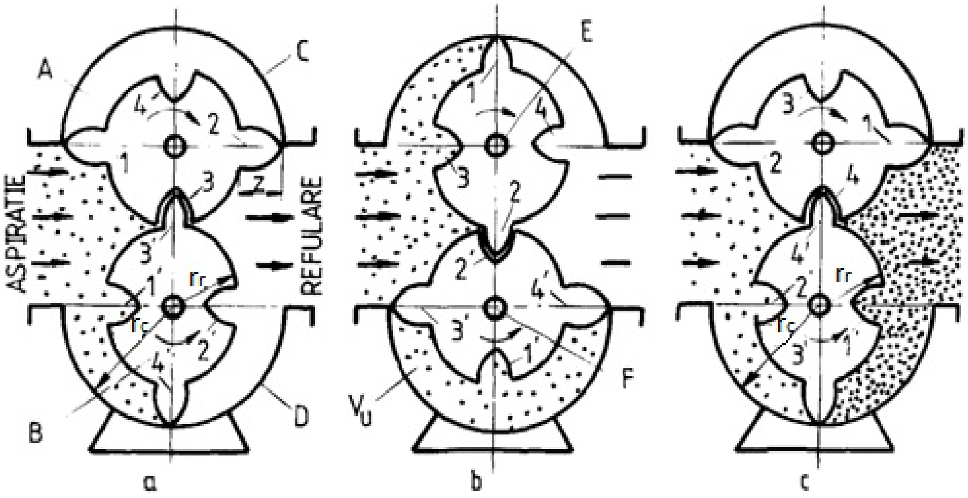

Figure 1: Rotating machine sketch: A, B - rotors; C, D – semi- cylinders; E, F - shafts; 1, 2, 3 ', 4' - rotating pistons; 1 ', 2', 3, 4 - gaps into which the rotating pistons enters

$$V_u = \pi l z\left(z + \pi \cdot r_r\right)\left[\mathrm{m}^3 / \mathrm{rot}\right] \qquad (2)$$

The fluid flow rate circulated by a rotor:

$$\dot{V} = \pi l z\left(2 \cdot r_r + z\right) \cdot \frac{n}{30}\left[\mathrm{m}^3 / s\right] \qquad (3)$$

For the entire compressor, which has two identical rotors, the flow rate will be:

$$\dot{V}_m = 2\dot{V} = \pi l z\left(2 \cdot r_r + z\right) \cdot \frac{n}{30}\left[\mathrm{m}^3 / \mathrm{s}\right] \qquad (4)$$

From the relation (4) it is noted that the volumetric flow rate linearly increases with the length, the rotor radius (l, r_r) and the rpm (n).

The necessary theoretical driving power of the machine is obtained from the relation [4] [5]:

$$P_m = \dot{V}_m \Delta p = \pi \cdot l \cdot z \cdot \left(2 \cdot r_r + z\right)\frac{n}{30} \cdot \Delta p \ \left[W\right] \quad (5)$$

From the relation (5) it is noted that at the increase of the pressure difference between suction and discharge (Δp), the driving power will increase.

3. The influence of the rotating piston shape on the machine circulated flow rate

The hypotheses are made:

- rotor length (l) and its radius (r_r) are constant;
- the ratio between the piston height (z) and the case radius (r_c) is constant z/r_c=ct;
- the machine rpm is constant;
- Δp is constant;

The rotating piston shape of length l, in section can be (Figure 2):

a) Rectangular blade on negligible thickness (theoretical case);

b) Triangle;

c) Curvilinear.

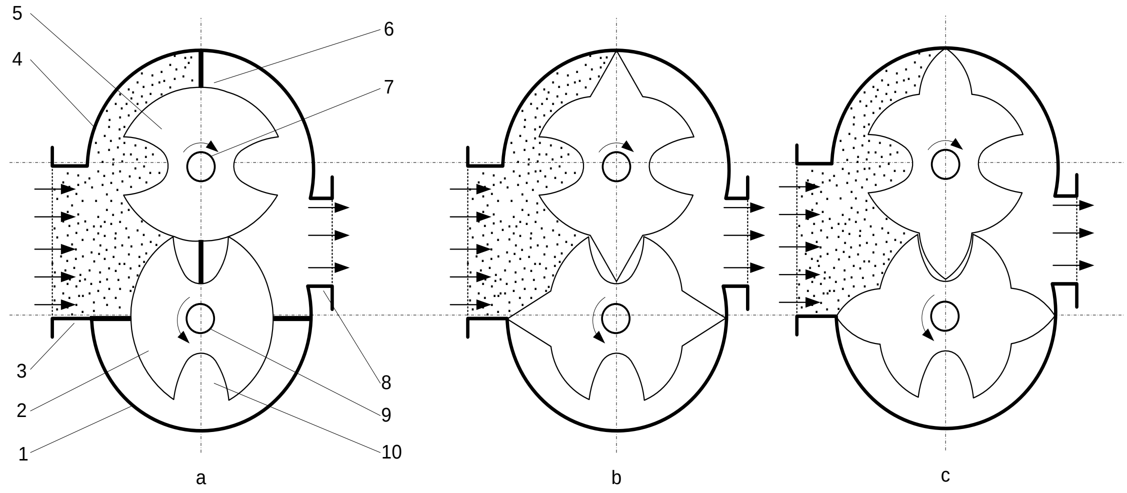

Figure 2: Section through the rotor in the three cases a, b, c
1 – lower case; 2 – lower rotor; 3 – suction chamber; 4 – upper case; 5 – upper rotor; 6 – rotating piston; 7 – driven shaft; 8 – discharge chamber; 9 – driving shaft; 10 – cavity where the upper rotor piston enters

The volumetric flow rate for the three cases is calculated.
By replacing the date's l = 0.05 m; z = 0.03 m; r_r = 0.05 m; n = 300 rot/min, in equation (4) is obtained:
a)

$$\dot{V}_m = \pi \cdot 0.05 \cdot 0.03 \left(0.03 + 2 \cdot 0.05\right) \cdot \frac{300}{30} \qquad (6)$$

$$\dot{V}_{m,a} = 0.006123\ m^3/s = 22.042\ \ m^3/h \qquad (7)$$

b) As shown in Figure 3 the useful volume V_u is reduced by the ABC and A'B'C' prisms volumes; they are equal and together gives the volume of a triangular section piston, namely, a prism with the dimensions: height: z = 30 mm; base b = 30 mm; length: l = 50 mm.

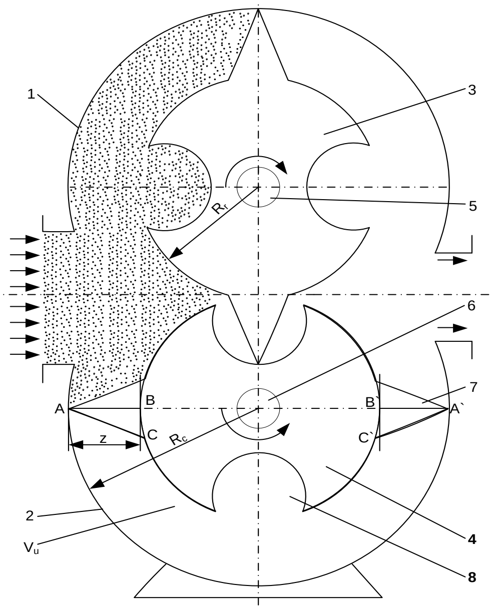

Figure 3: Section through the rotating working machine.
1- upper case; 2-lower case; 3-upper rotor; 4-lower rotor; 5,6-shafts; 7-triangular piston; 8-cavity

The area of the section between the basis of the prism and the rotor is neglected. The volume of this prism will be:

$$V_{p,II} = A_{bazei} \cdot l = \frac{1}{2} \cdot b \cdot z \cdot l = \frac{1}{2} \cdot 0.03 \cdot 0.03 \cdot 0.05 = 0.0225 \cdot 10^{-3} \ m^3 / rot \quad (8)$$

Compared with the theoretical flow rate transported by the machine in the version a):

$$\dot{V}_{m,b} = \pi l z (z + 2r_r) \cdot \frac{n}{30} \quad [m^3 / s] \quad (9)$$

the theoretical flow rate of the machine in this version b) will be reduced with V_{pII}.
The gas flow rate transported by a rotor:

$$\dot{V}_u = \left[\pi l z \left(z + 2r_r \right) - V_{p,II} \right] \left[\mathrm{m}^3 / \mathrm{rot} \right] \quad (10)$$

The machine has two identical rotors, so that the flow rate transported will be:

$$\dot{V}_{m,b} = 2 \cdot \left[\pi l z (z + 2r_r) - \frac{1}{2} b z l \right] \cdot \frac{n}{60} \quad [m^3 / s] \ (11)$$

For the same data as in version a) and adding b=0.03 m, we obtain a flow rate of:

$$\dot{V}_{m,b} = \left[\pi \cdot 0.05 \cdot 0.03 \left(0.03 + 2 \cdot 0.05 \right) - \frac{1}{2} \cdot 0.03 \cdot 0.03 \cdot 0.05 \right] \cdot \frac{300}{30} \quad (12)$$

$$\dot{V}_{m,b} = 0.0061219 \ m^3 / s = 22.038 \ m^3 / h \quad (13)$$

c) It can be remarked from figure 4 that the useful volume V_u will be reduced with a volume equal to the areas (*ABC+A'B'C'*) multiplied with the length of the piston (l), dimension perpendicular on the plane of the figure. The areas ABC and A'B'C' are equal; hence the volume of a curvilinear piston ($V_{p,III}$) must be subtracted from V_u.

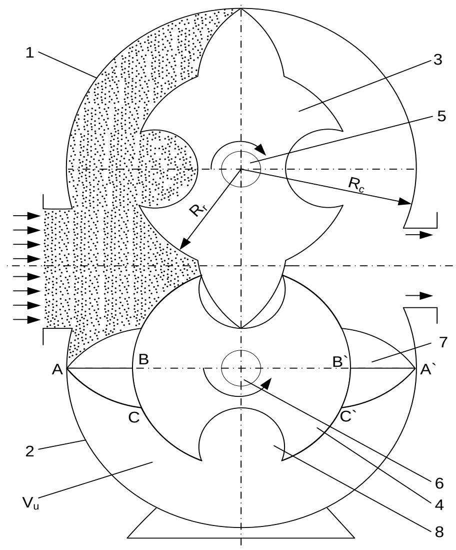

Figure 4: Transversal section through the rotating working machine
1-upper case; 2-lower case; 3-upper rotor; 4-lower rotor;
5,6-shafts; 7-curvilinear piston; 8-cavity

This volume equals the piston area (S_p) multiplied by its length (l).

$$V_{p,III} = S_p \cdot l = 2 \cdot S_{ABC} \cdot l \quad [m^3] \quad (14)$$

Hence the theoretical machine flow rate will be reduced with $V_{p,III}$ leading to:

$$\dot{V}_u = \left[\pi l z \left(z + 2r_r\right) - \dot{V}_{p,III}\right] \quad [m^3/rot] \qquad (15)$$

The flow rate transported by a rotor:

$$\dot{V}_u = \left[\pi l z \left(z + 2r_r\right) - \dot{V}_{p,III}\right] \cdot \frac{n}{60} \quad [m^3/s] \qquad (16)$$

Because the machine has two identical rotors, the volumetric flow rate of the machine will be:

$$\dot{V}_{m,c} = 2\dot{V}_u = \left[\pi l z \left(z + 2r_r\right) - 2S_{ABC} \cdot l\right] \cdot \frac{n}{30} \quad [m^3/s] \quad (17)$$

The value of this volume was calculated in [6] and has result:

$$\dot{V}_{m,c} = 0.005725 \quad m^3/s = 20.613 \quad m^3/h \qquad (18)$$

It is noted that the piston shape influences the flow rate value, namely:

$$\dot{V}_{m,a} > \dot{V}_{m,b} > \dot{V}_{m,c}$$

4. The influence of the rotating piston shape on the driving power of the machine

Depending on the constructive solution of the rotor, the power lost through viscous friction between the rotors and the side walls of the case will be different; as a result and the necessary driving power of the machine will be different.

There are studied two alternatives:

a) version I, the rotor has a constant thickness;
b) version II, when the rotor is specially processed.

a) The determination of lost power through viscous friction (version I)

In this version (Figure 5.) the entire front surface of the rotors comes in contact with the side walls of the case.

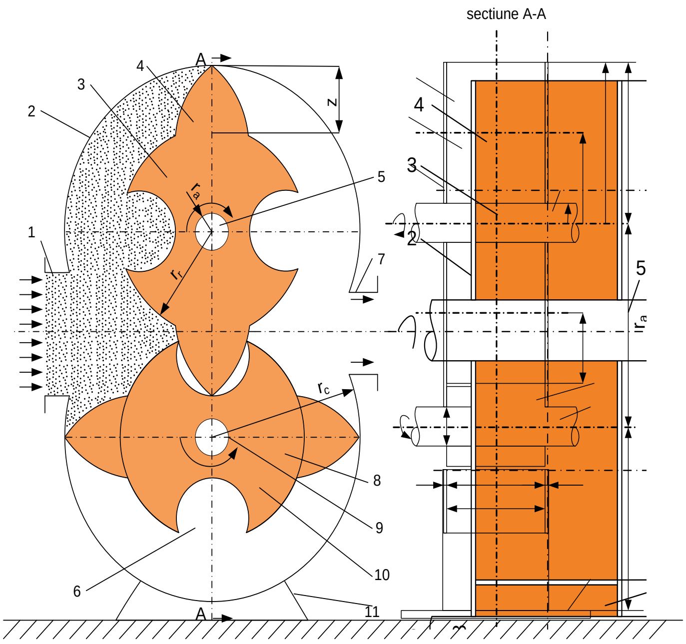

Figure 5: Cross section (a) and longitudinal section (b) through the rotating compressor 1-gas suction connection; 2-upper case; 3-upper rotor; 4- rotating piston; 5-driven shaft; 6-cavity; 7-gas discharge connection; 8-lower rotor; 9-driving shaft; 10-contact surface between the rotor and the case wall; 11-support

Figure 6: Axonometric view of the rotors with constant thickness (version I)

The aim is to determine the power consumed by viscous friction between the frontal surfaces of the rotors and the case walls.

Are known (Fig. 7):

- The shaft Radius on which the rotor is mounted r_a= 9 mm; - the rotor external radius r_r= 50 mm; - the angular velocity of the disc, if the rpm is n = 200, 400, 600, 800, 1000 rot/ min; - the dynamic viscosity at t = 20°C for air; - the gap between disc and the case wall (s) is selected with the CNC processing precision.

Calculation hypothesis:

- the friction forces between the piston head and the case are neglected.
- the front surface of the piston completes the cavities created in the rotor; the calculation area is between two circles with the rays r_a and r_r.
- the fluid velocity at each point of the front surface of the rotor is equal to the rotor velocity.

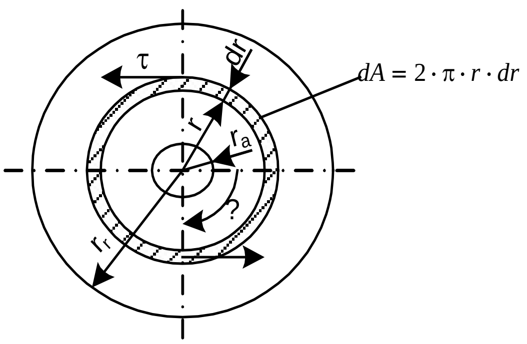

Figure 8: Plan view of the rotor

$$dA = 2\pi r \cdot dr \qquad (22)$$

The tangential tension (shear stress) due to the fluid viscosity is computed with Newton relation [8][9]:

- the velocity at a point on the disc will be in the range:

$$w_a = \omega \cdot r_a \text{ and } w_r = \omega \cdot r_r \quad [m/s] \qquad (19)$$

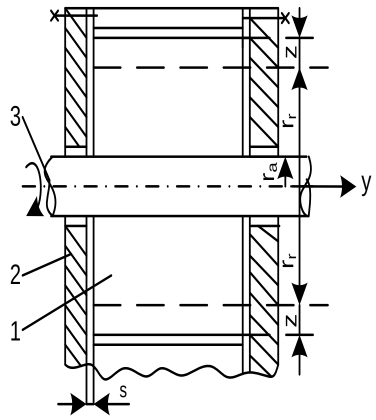

Figure 7: Computing notations
1 rotor; 2 case; 3-shaft

In Figure 7 has been noted r_a- shaft radius; r_r- rotor radius; z- piston height.

The tangential velocity at each point of the rotor is given by: w=ω·r [m/s].

The calculation is performed for a rotor. From mechanic it is known that torque is the product between force and the arm force. The elementary resistant torque due to the viscous friction between the rotor and the two walls of the case will be [7]:

$$dM_r = 2rdF_f \qquad (20)$$

where F_f is the viscous friction force.

$$V_u = \left(\pi \cdot r_c^2 - \pi \cdot r_r^2\right) \cdot l \left[\mathrm{m}^3 / \mathrm{rot}\right] \qquad (21)$$

τ- the tangential effort;
dA- the elementary surface area (Fig. 8).

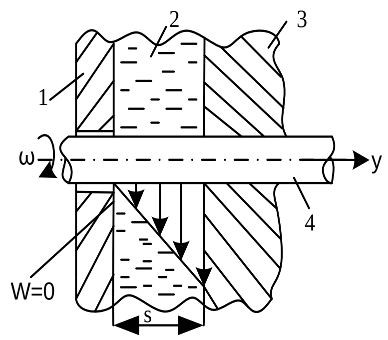

Figure 9: Computing section
1 case; 2 thin layer of fluid; 3- rotor disc; 4 - shaft

$$\tau = \eta \frac{dw}{dy} \qquad (23)$$

where the coordinate y is measured perpendicularly to the disk surface (Figure 9).

The velocity gradient for the boundary layer with "s" thickness, assuming a linear variation, has the expression:

$$\frac{dw}{dy} = \frac{\omega r}{s} \quad (24)$$

Equation (23) becomes:

$$\tau = \eta \cdot \frac{\omega r}{s} \quad (25)$$

Equation (21), taking into account equation (22) and (25) becomes:

$$dF = \eta \cdot \frac{\omega \cdot r}{s} \cdot 2\pi r dr = 2\pi r^2 \eta \frac{\omega}{s} dr \quad (26)$$

Equation (20) becomes:

$$dM_r = 2r \cdot 2\pi r^2 \eta \frac{\omega}{s} dr \quad (27)$$

$$\int_0^M dM_r = \int_{r_a}^{r_r} \frac{4 \cdot \pi \cdot \omega \cdot \eta \cdot r^3}{s} dr \quad (28)$$

$$M_r = \frac{\pi \cdot \eta \cdot \omega \cdot r_r^4}{s} - \frac{\pi \cdot \eta \cdot \omega \cdot r_a^4}{s} \quad (29)$$

$$M_r = \frac{\pi \cdot \eta \cdot \omega}{s}[r_r^4 - r_a^4] \; [N \cdot m] \quad (30)$$

From mechanics is known the relation of consumed power computing to overcome the viscous friction for one rotor [7]:

$$P_{1r} = M_r \cdot \omega \; [W] \quad (31)$$

For the entire machine, the consumed power by viscous friction (P_m) will be:

$$P_m = 2 \cdot P_{1r} \; [W] \quad (32)$$

From (30) and (31) it results:

$$P_m = 2 \cdot \frac{\pi \cdot \eta \cdot \omega^2}{s}\left[r_r^4 - r_a^4\right] \; [W] \quad (33)$$

From (30) it is found that the dynamic viscosity (η) has a great influence on viscous friction power losses.

A more exactly calculation can be performed using the dynamic boundary layer theory [10][11]; the tangential effort at several points in the boundary layer section is determined and the calculation of F_f is performed with an average value.

• The lost power by viscous friction between the frontal surfaces of the rotors and the case walls with values of η and ω for air and a given n = 200 rpm, is calculated.

$$\omega_1 = \frac{2\pi n_r}{60} = \frac{\pi}{30} 200 = 20.93 \; rad/s$$

For air [12]:

$\eta_{aer} = 18.5 \cdot 10^{-6} \; \frac{N \cdot s}{m^2}$ and form [13] $s=0.01 \cdot 10^{-3}$m:

$$P_{m,1} = 2 \cdot \frac{3.14 \cdot 18.5 \cdot 10^{-6} \cdot 20.93^2}{0.01 \cdot 10^{-3}} \left[\left(50 \cdot 10^{-3}\right)^4 - \left(9 \cdot 10^{-3}\right)^4\right] = 0.031 \, W$$

Similarly ω is calculated for n = 400, 600, 800, 1000 rpm and subsequently the lost power by viscous friction, resulting the values shown in Table 1.

Table 1. Values of ω and P_m

n_r [rot/min]	200	400	600	800	1000
ω [rad/s]	20.93	41.86	62.80	83.73	104.66
P_m [W]	0.031	0.127	0.286	0.508	0.791

Based on the results in Table 1, for air, the curve P_m = f (n_r) is plotted:

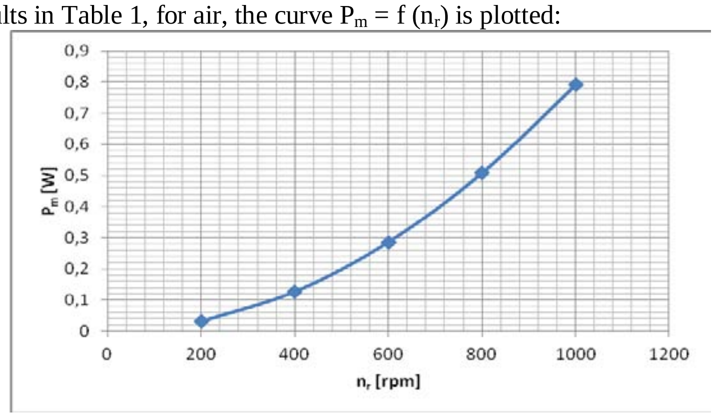

Figure 10: P_m = f (n_r) for air (version I)

b) The determination of lost power through viscous friction (version II)

In this version (Fig. 11), not all of front surface of the rotors comes in contact with the side walls of the case.

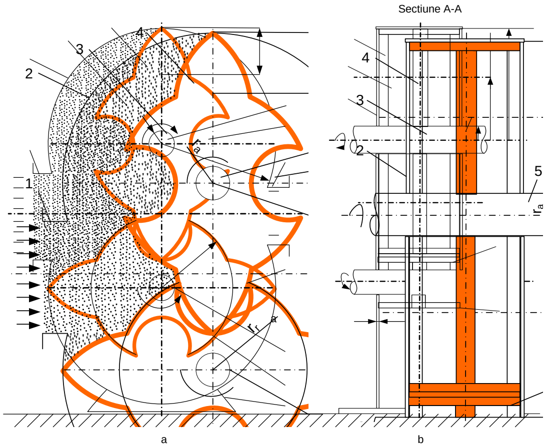

Figure 11: Cross section (a) and longitudinal section (b) through the rotating compressor
1-gas suction connection; 2-upper case; 3-upper rotor; 4- rotating piston; 5-driven shaft; 6-cavity; 7-gas discharge connection; 8-lower rotor; 9-driving shaft; 10-contact surface between the rotor and the case wall; 11-support

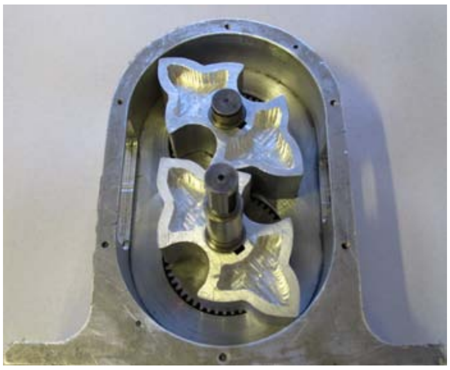

Figure 12: Axonometric view of the rotors (version II) mounted in the case

Similarly to a) the power consumed by friction between the two rotors and the side walls of the case is:

$$P_m = 2\cdot\frac{\pi\cdot\eta\cdot\omega^2}{s}\left[r_r^4 - r_i^4\right]\ [N/m]\quad(34)$$

where r_i is the inner radius of the rotor; r_i = 45 mm = 45 · 10^{-3} m;

For air, and the same values of η, s and r_r as in version I, is obtained:

$$P_m = 2\cdot\frac{3.14\cdot18.5\cdot10^{-6}\cdot(20.939)^2}{0.01\cdot10^{-3}}\left[\left(50\cdot10^{-3}\right)^2-\left(45\cdot10^{-3}\right)^2\right]=0.010\ W \qquad (35)$$

Similarly, the calculation for n_r = rot/min 400, 600, 800, 1000, are made, resulting the data in Table 2.

Table 2. Values of P_m=f(n_r) for version II

n_r [rot/min]	200	400	600	800	1000
ω [rad/s]	20.933	41.866	62.799	83.680	104.660
P_m [W]	0.010	0.043	0.098	0.174	0.273

Based on the results in Table 2, the curve P_m = f (n_r) is plotted in Figure 13:

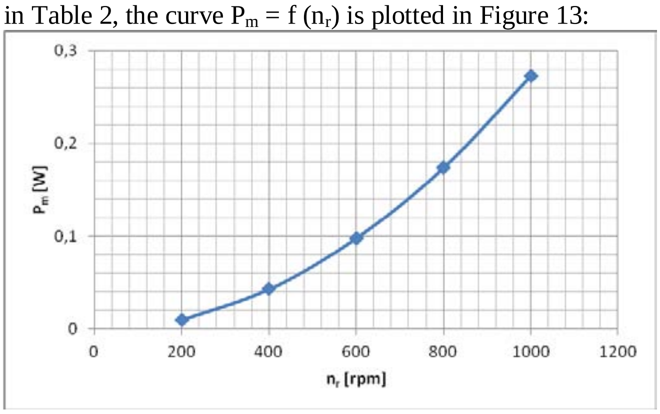

Figure 13: P_m = f (n_r) for air (version II)

If overlap on the same drawing the graphs from Figure 8 and form Figure 11, Figure 14 is obtained:

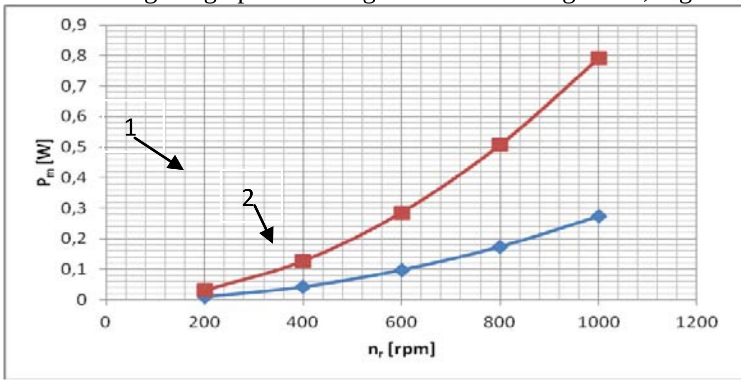

Figure 14: $P_m = f(n_r)$ for air
1- $P_m = f(n_r)$ version I; 2- $P_m = f(n_r)$ version II

From Figure 14 it is found that with the machine rpm increase, the power consumed by viscous friction between the machine rotors and the case the will increase; the machine constructive solution in version II will be more advantageous because the power consumed to defeat the viscous friction is lower (Figure 14).

5. Conclusions

From the material presented above the following conclusions results:

1. For a certain rotor dimension (r_r, l), the rotating piston shape influences the gas flow rate circulated by the compressor.

2. When the rotating machine will function as the pump, the triangular section of the rotating piston is selected; when operating as fan the curvilinear section is selected, because it provides a better sealing.

3. The power lost by viscous friction between the frontal surfaces of rotors and the case wall is higher in version I than in version II.

4. The driving power of the machine (shaft required power) will be lower in version II because here, the viscous friction between the rotor and the case occurs on smaller surfaces than in version I.

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