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*Z-transform solution for nonlinear difference equations*

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| **A r t i c l e I n f o** | Abstract |
| ReceivedAccepted | The aim of this paper is to study Z-transform to solve non-linear difference equations, after converting them to linear difference equations by one of the conversion methods. This is because the z-transform cannot be directly applied to the nonlinear difference equations.Key words: difference equations , nonlinear difference equations , Z-transform.  |
| **الخلاصـة** |
| الهدف في هذا البحث هو دراسة تحويل زد لحل معادلات الفروق الغير خطيه ،وذلك بعد تحويلها الى معادلات الفروق الخطية بأحدى طرق التحويل. وذلك لأنه لا يمكن تطبيق تحويل زد مباشرتا على معادلات الفروق غير الخطية 0  |

## Introduction*)*

Transformation is a very powerfur mathematical tool so using it in mathematical treatment of problem is arising in many application .The idea of Z-transform back to 1730 when De Moivre introduced the concept of "generating functions" to probability theory[14]. in 1947 a transform of sampled signal or sequence defined by W. Hurewicz as a tractable way to solve linear difference equations. The transformation named "Z-transform" by Ragazzini and Lotfi Zadeh in the sampled-data control group at Columbia University in 1952. Z-transform is transformation for discrete data equivalent to the Laplace transform of continuous data and it’s a generalization of discrete Fourier transform[12] .

Z-transform is used in many areas of applied mathematics as digital signal processing, control theory, economics and some other fields[13].

Difference equation are models of the world aroundus[3,5]. From clocks to computers to chromosomes ,processing discrete objects in discrete steps is acommon theme, and are the discrete equivalent of differential equations and arise whenever an independent variable can have only discrete values[2]. The Difference equations are used in situations of real life, in various sciences population models, genetics, psychology, economics, sociology, stochastic time series, combinatorial analysis, queuing problems, number theory, geometry, radiation quanta and electrical networks.

Recently there has been a great interest in studying nonlinear difference equations [1]and one of the reasons is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real-life situations in population biology, economy, probability theory, genetics, psychology, sociology, and so forth. Some nonlinear difference equations, especially the boundedness, global attract, oscillatory and some other properties of second order nonlinear difference equations have been investigated by many authors see [4,10,11]. We need to study nonlinear equations of difference because almost all biological processes are nonlinear. In this paper we use the Z- transformation to solve nonlinear difference equations. This paper consists of several sections, the first section includes the introduction, the second section contains Z-transfer and the third section includes the conversion of nonlinear difference equations into linear difference equations.

2.Z-transform

Definition:

 Let be a sequence of numbers such that, The Z-transform of this sequence is the series Where Z is the transform variable. **A Table of Properties of Z-transform**

|  |  |  |
| --- | --- | --- |
| N | Sequence | Z-transform |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |

Figure 1: Organization of the paper

1. Converting nonlinear difference equations into linear difference equations.

There are several ways to convert :

The first form of nonlinear difference equationhomogeneous in [7] can be expressed in the following form:

If the nonlinear function is a polynomial function of then equation (a.1) can be written as

Where is a known function of and is the order of the polynomial function of The solution to each of the linear equations

Provide a solution to equation (a.1).

The second formof nonlinear difference equations

Consider the special class of nonlinear difference equations[9]

Where the are constants and is a given function. This nth-order ,nonlinear equation can be transformed into an nth-order linear equation.

We can solving equation (b.1) by taking the logarithm of equation:

And define

Thus , satisfies the following linear , inhomogeneous nth-order equation with constant coefficients:

c. The third form of nonlinear difference equations is of the Riccati type [8].

Let Direct substitution of this expression into Equation(c.1) yields the linear equation

Through this substitution we obtain the equation of the linear differences that can be solved using the Z-transform.

Then solutions of Equation (c.1) are obtained from the relationship between and

d. The four form of nonlinear difference equations

this technique is based on Lie's transformation group.

To convert equation(d.1) into a linear equation, we follow the following

Let's begin by assuming that a solution of the functional equation

Is known for some constant . Then we define a new dependent variable by

For belonging to an open interval in which is different from 0.

Using the chain rule , we have

Now we integrate to obtain

Or

Which is a linear equation of first order with constant coefficients.

After the conversion, we can apply Z-transform, as shown in the following examples.

**Application:**

Example 1. Consider the nonlinear difference equation

Now, we will convert the nonlinear difference equation to linear difference equation

We are going to take and plug it into the equation (E.1)

 Or

We will substitute the equation (E.3) and (E.4) into equation (E.2)

Or

Or

the equations (E.5)and (E.6)are linear difference equations[6]

We will take Z-transform of equations

Where

We will take Z-transform of equations

Where

Example 2. Consider the nonlinear difference equation

Now, we will convert the nonlinear difference equation to linear difference equation

If we set then satisfies the

Where

Now, can be taking Z-transform of

Since

Example 3. Consider the nonlinear difference equation

Let

We will substitute the (E.2) into the equation (E.1)

Where

Now, can be taking Z-transform of

The general solution of the Riccati equation is

Where C is arbitrary.

Example 4. Consider the nonlinear difference equation

Now, we will convert the nonlinear difference equation to linear difference equation

We are going to take and plug it into the equation (E.1)

 Or

We will substitute the equation (E.3) and (E.4) into equation (E.2)

Or

Or

the equations (E.5)and (E.6)are linear difference equations

We will take Z-transform of equations

Where

We will take Z-transform of equations

Where

Example 5. Consider the nonlinear difference equation

Where a is a constant and

From equation

We have

The from of this last equation suggests that we try a linear expression for say,

 We obtain

Equation coefficients leads to

 and

Let and from equation

So we take

Or

Now we substitute the last expression into

To obtain

Or

Or

Where

We will take Z-transform of equations

And finally

Example 6. Consider the nonlinear difference equation

Now, we will convert the nonlinear difference equation to linear difference equation

If we set then satisfies the

Where

Now, can be taking Z-transform of

Since

## Conclusion

In this paper we discussed how to solve nonlinear difference equations using Z-transform, but it turns out that nonlinear difference equations cannot be solved directly by Z-transform.

In this paper, we present methods for converting nonlinear difference equations into linear difference equations that can be solved directly by Z transformation.

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