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The Effects of Lift Curve Slope and Lift Coefficient on the Wing Cessna 172S

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Abstract. The main idea of this paper is to give a profound investigation of the aerodynamic properties of a wing for Cessna 172S. The numerical analysis of total drag is developed by using various methods from different references, the 2D and 3D lift curve parameters of the wing Cessna 172S are generated according to the analysis method depending on empirical equations. The first stage of modelling Cessna 172S is evaluated according to the airfoil of Cessna NACA 2412, whereas the second stage of modelling is estimated by using wing planform properties. The maximum lift coefficient, lift curve slope, zero angle of attack, lift at zero angle of attack, and stall angle of attack are calculated by using different empirical equations.

Keywords: empirical methods, DATCOM method, Cessna 172S, lift curve parameters, Theory of wing section.

Nomenclature

AR	=	Aspect ratio	$\Lambda_{C/4}$	=	Sweep angle of quarter chord
b	=	Wingspan	α_0	=	Zero angle of attack
c	=	Chord length	α	=	Angle of attack
c _r	=	Root chord	α_i	=	Angle of attack for design lift
c_t	=	Tip chord	α_{CLma}	~ =	Maximum lift angle of attack
CD	=	Drag coefficient	α_{stall}	- =	Stall angle of attack
C _{Di}	=	Lift-induced drag coefficient	β	=	Mach number parameter
C_{Dmin}	=	Minimum drag coefficient	δ	=	Induced-drag factor
CI	=	Two-dimensional lift coefficient	λ	=	Taper ratio
C_{LminD}	=	Lift coefficient at drag minimum	Г	=	Dihedral angle
C_L	=	Three-dimensional lift coefficient	C_{10}	=	Lift at zero angle of attack
C_{Lmax}	=	Maximum 3D lift coefficient	e	=	Oswald efficiency factor
C_{Imax}	=	Maximum 2D lift coefficient	k	=	Lift -induced drag factor
$C_{l\alpha}$	=	2D lift curve slope	Κ	=	Empirical correction factor
$C_{L\alpha}$	=	3D lift curve slope	K۸	=	Sweep correction factor
R _{L.S}	=	Lifting-surface correction factor	L	=	Lift force
Re _c	=	Reynold number at mean chord	М	=	Mach number
Re	=	Reynold number	Amar	=	Sweep angle of maximum
\mathbf{S}_{wet}	=	Wetted reference area	thickn	ess	2
S _{ref}	=	Wing reference area	C _D	=	The zero lift-drag coefficients
$\left(\frac{t}{c}\right)$	=	Thickness ratio	$\Lambda_{C/2}$	=	Sweep angle of half chord
V	=	Airspeed of aircraft	$\Lambda_{\rm LE}$	=	Sweep angle of leading edge
W	=	Maximum weight of aircraft	$\Delta C_{L ma}$	_{1x} =	Mach number correction

INTRODUCTION

The aerodynamic designers discovered the theoretical and practical ways to predict drag since the first time from discovering aeronautics. Also, these ways gave the results from empirical methods such as Hoerner's drag estimation methods[1] and lifting line theory from Prandtl [2]. In this study, the classical methods will be used to predict the value of the drag based on the geometry of aircraft and flow conditions. The coefficients for estimating drag they are fundamental with aerodynamic forces and moments, and the coefficient from this process represents the total drag of

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aircraft such as drag model. The drag model is a mathematical expression of the drag coefficient, which depicts how the drag of aircraft or wing changes as the function of the flow field [3]. The total drag of aircraft can be divided into two different types in subsonic of aircraft: zero-lift drag or parasite drag and (induced drag or drag due to lift). The zero-lift drag can be divided to the skin-friction drag, (pressure drag or form drag) and additional profile drag due to lift (drag from 2D airfoil section):

Total Drag = Parasite Drag(C_{D_0}) + Induced Drag (C_{D_I})

The geometry of aircraft has been presented as Cessna 172S. The airfoil of Cessna has adjusted such as NACA 2412 to estimate 2D prediction of aerodynamic properties for. Various methods have used to estimate 3D prediction of aerodynamics characteristic of Cessna 172S such as lift curve slope, maximum lift coefficient and angle of attack by using DATCOM method and multi references. The sweep angle of Cessna 172S and mean aerodynamic chord has been estimated by using the geometrical method and theoretical analysis. The previous study of the developing of aerodynamic model[25] shows the difference between the lift curve slope by using the same equations for another wingspan and aircraft.

In short, the purpose of this study is to discuss and estimate many methods to calculate the total drag coefficient such as zero-lift drag coefficient, skin-friction drag coefficient, and induced drag coefficient. Investigate the drag characteristics of the wing to predict the total drag of wing and present a different method from various sources such as DATCOM methods[4] to calculate the lift curve slope of the wing NACA 2412 of Cessna 172S.

Lift Curve Slope Prediction

The lift curve slope is a necessary parameter to produce and generate the aerodynamic model such as Cessna 172S. The lift curve must be estimated in 3 stages; 2D wing airfoil, 3D wing and the last one for entire aircraft. According to the Equation (2) [3] (linear range), the lift curve slope can be defined by using four parameters: zero-lift angle of attack, the angle of attack, maximum lift coefficient, and angle of attack of lift coefficient.

$$C_L = C_{Lo} + C_{L\alpha}\alpha \tag{2}$$

2D Lift Curve Slope

The $C_{l\alpha}$ can be calculated by using DATCOM [4] for arbitrary airfoils by using Equation 3.

$$C_{l\alpha} = \frac{1.05}{\beta} K (C_{l\alpha})_{\text{Theory}}$$
(3)

Where β is the compressibility correction factor $\beta = (1 - M^2)^{0.5}$, K is empirical correction factor $\frac{C_{l\alpha}}{C_{l\alpha \, theory}}$ and it can be estimated by using Figure (1) and (C_{la}) theory can be predicted by using Figure (2)



FIGURE 1. Geometry Factor K vs Trailing Edge

As can be seen in Figure (1) the magnitude of trailing edge angle can be expressed by using $tan \frac{1}{2} \phi_{TE} = \frac{\frac{1}{2}(Y_{90} - Y_{99})}{9}$ (4) Where Y₉₀ and Y₉₉ are the airfoil thicknesses at 90% and 99% of the chord back from the leading edge, and it can

be predicted from the data of DATCOM [6] [4].



FIGURE 2. (Cl α) theory vs. Wing thickness ratio t/c

3D Lift Curve Slope

The 3D lift-curve slope can be estimated by using lifting line theory according to the formula from Anderson [7,21]

$$C_{L\alpha} = \frac{C_{l\alpha}}{1 + \frac{C_{l\alpha}}{\pi AR} (1 + \delta)}$$
(5)

Another method to predict lift-curve slope is obtained from Helmbold [8,23] according to the formula.

$$C_{L\alpha} = \frac{2\pi AR}{2 + \sqrt{AR^2 + 4}} \tag{6}$$

As can be shown in Figure (3) Polhamus has obtained another method to predict lift-curve slope according to Equation (7) which is obtained from NACA TR-3911[9], and it is presented by DATCOM [4].

$$C_{L\alpha} = \frac{2\pi AR}{2 + \sqrt{\left(\frac{AR\beta}{k}\right)^2 \left(1 + \frac{tan^2 \Lambda_{C/2}}{\beta^2}\right) + 4}}$$
(7)



Maximum Lift Coefficient Prediction

The biggest value of the lift coefficient can express by $C_{L max}$, two stages can show the maximum lift coefficient; 2D and 3D lift coefficient. Also, it is changing with many parameters such as; airfoil section, the location of maximum thickness, camber, Mach and Reynold number.

2D Maximum Lift Coefficient Prediction

The maximum lift coefficient can be estimated by using the database of the airfoil or experimental data depending on the Mach and Reynold's number such as the data from Von Doenhoff and Albert Edward [10].

Another method to predict fast estimation for maximum lift coefficient is obtained from Raymer [11] by using Equation (8) [11] [3]

$$C_{l \max} = (C_{l \max})_{root} + \frac{2 y_{MGC}}{b} \left[(C_{l \max})_{tip} - (C_{l \max})_{root} \right]$$
(8)

Where y_{MGC} is the distance, from the root chord to the mean geometric chord and it can be estimated according to Equation (9) which is obtained from DATCOM[4] for general wing of aircraft in addition $(C_{lmax})_{root}$ and $(C_{lmax})_{tip}$ can be predicted by using the Xfoil -data[12] or database of aircraft from Doenhoff and Albert Edward [10].

$$y_{MGC} = \frac{2}{s} \int_0^{b/2} cy \, dy \tag{9}$$

3D Maximum Lift Coefficient Prediction

Maximum lift coefficient for three-dimensional wing can be estimated by using Equation (10), which is obtained from Raymer [11]. This method was not an accurate method but still applicable to calculate maximum lift coefficient to the fast estimation for the designer, and it should be replaced with methods that are more accurate.

$$C_{L_{max}} = C_{L_{max0}} \times K_A \tag{10}$$

Where $C_{L max0}$ is the maximum lift coefficient of the upswept wing and it can be calculated from Equation (11), which is obtained from Gudmundsson [3], and K_{Λ} is the sweep correction factor, and it can be predicted from Raymer[11] and Jenkinson[13] according to Figure (4) or from Young[14,23]

$$C_{L_{max0}} = 0.9 \times C_{l\,max} \tag{11}$$

Where $C_{l max}$ is the 2D maximum lift coefficient.



FIGURE 4. Determination Sweep correction factor for CLmax.

DATCOM [4] has obtained another method to predict 3D max lift coefficient for the subsonic upswept wing for high aspect ratio by using Equation (12).

$$C_{L_{max}} = \left(\frac{c_{L_{max}}}{c_{l max}}\right) C_{l max} + \Delta C_{L_{max}}$$
(12)

Where $C_{L max}/C_{l max}$ is ratio obtained from Figure (5), $C_{l max}$ is 2D lift max coefficient, and $\Delta C_{L max}$ is Mach number correction, and it is equal zero for Cessna 172S.

As can be shown in Figure (5) the Δy is a parameter of leading edge, and it can be calculated by using the formula $\Delta y = 25$ (t/c) for NACA 4 digits' airfoil.



FIGURE 5. CLmax/Clmax ratio data plot based on DATCOM .

Another formula to predict 3D max or steady level lift coefficient is obtained from Roskam [15] and Gudmundsson [3,20] :

$$C_L = \frac{2W}{\rho V^2 S} \tag{13}$$

Where W is a maximum weight of aircraft, V is the velocity, S is the wingspan of aircraft and ρ is the density

Zero-Lift Angle of Attack Prediction α₀

No lift is going to be at the zero-lift angle of attack, which it can be predicted for 2D from the database or experimental data of the airfoils such as NACA TR-824[16] reports or Von Doenhoff and Albert Edward[10,22].

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Another method to predict two-dimensional zero-lift angle of attack for the airfoil is obtained from DATCOM[4] by using Equation (14)[4]. This method using for airfoil which does not have a database and it is based on the table from DATCOM (4-1-1-D)[4].

$$\alpha_0 = k \left(\alpha_i - \frac{C_l}{C_{l\alpha}} \right) \tag{14}$$

Where k is an empirical factor depending on the airfoil series, α_i is the angle of attack for design lift, and it can be predicted from Table 4-1-1-D by DATCOM[4], $C_{l\alpha}$ is lift-curve slope = 2π and C_1 is 2D lift coefficient. For untwisted wing and subsonic DATCOM obtained another method but without empirical factor by using 2D or 3D lift curve slope:

$$\alpha_0 = \left(\alpha_i - \frac{c_l}{c_{l\alpha}}\right) \tag{15}$$

From this formula (15), the lift coefficient can be predicted by using Equation (16)[17]. $C_l = C_{l\alpha}(\alpha_i - \alpha_0)$ (16)

Stall Angle of Attack Prediction

The stall angle of attack for wing of Cessna 172 can be predicted according to Equation (17) which has obtained from DATCOM [4].

$$\alpha_{stall} = \frac{c_{L_{max}}}{c_{La}} + \alpha_0 + \Delta \alpha_{stall} \tag{17}$$

Where $C_{L\alpha}$ lift curve slope for the wing, α_{ZL} is zero lift angle and $\Delta \alpha_{stall}$ is correction factor.

The Results of Lift Curve Slope Prediction

The lift curve parameters have calculated according to the database of Cessna 172S which can be shown in Table (1)

Data of Cessna 172S				
AR	7.52			
$\mathbf{S}_{ ext{wet}}$	32.967			
t/c	0.12			
b	10.922			
C_r	1.6256			
C_t	1.1303			
λ	0.672			
Γ	2.73°			
Airfoil	NACA 2412			
max weight of Cessna 172S	1111 kg			
The sweep of max thickness line	3.89			
sweep angle of leading edge Λ_{LE}	6.4020			
Sweep angle of quarter chord $\Lambda_{C/4}$	5			
Sweep angle of half chord $\Lambda_{C/2}$	3.513			
Sweep angle of trailing edge	7.5			
Умдс	2.701			

TABLE 1. The database of Cessna 172S.

The lift curve slope of the wing of Cessna 172S has predicted by various methods depending on the Oswald efficiency factor, aspect ratio and sweep angle of aircraft. It has estimated according to the lifting line theory, Roskam [15], Raymer[11], Helmbold[8] and Polhamus by using Equation [5, 6, 7] and Figure (5) demonstrated the comparison between these methods by using a different aspect ratio. As can be shown in T the lift curve slope has calculated by using different Oswald efficiency and velocity that obtained from Grosu [18], E. Obert [19] as well. According to these results, we see the values of Helmbold, John G and lifting line theory are approached to each other at the stall speed of aircraft and aspect ratio of Cessna 172S. The value of max lift coefficient has predicted by using the general formula according to the Roskam [15], And as can be shown in Figure (6) the magnitude of max lift is increasing with the stall speed of Cessna and decreasing with max-speed of aircraft depending on the max weight of Cessna 172S.

The Results of Lift Curve Slope 2D Prediction

The airfoil of Cessna 172 is NACA 2412, and according to the airfoil, the 2D lift curve parameters has predicted according to the Von Doenhoff and Albert Edward, DATCOM [4] Equation 3.5 and Gudmundsson. As can be shown in Table 2 the parameters of 2D lift curve slope have calculated carefully according to the references and numerical estimation.

Lift curve parameters	Von Doenhoff and Albert Edward	DATCON	М	Gudmundsson[3]	Ray mer
Lift curve slope $C_{l\alpha}$	6.223	5.503	M=0.074	5.7873	6.22 3
		5.622	M=0.216		
Maximum lift coefficient C _{1 max}	1.63	1.65		1.62	1.72
Zero-lift angle of	-2.74	-2		-2	-
attack α_0					2.74
Angle of attack for C ₁	16.1	16.8		16	16.8
max	0.205			0.202	0.22
Lift at zero angle of	0.295			0.202	0.25
Stall angle of attack for wing				16	+

TABLE 2. The 2D lift curve slope parameters estimation

According to Table 2, the magnitude of lift curve slope by Raymer and Albert Edward [5] is the same, Whereas the magnitude of lift curve slope by DATCOM [4] in Figure 6 has calculated according to the various Mach number and velocity by using the Equation . The value of maximum lift coefficient seems close to each other according to the Albert Edward [5], DATCOM [4], and Gudmundsson [3]. Whereas the magnitude of $C_{1 max}$ has calculated by using Equation 3.10 according to Raymer [15] where Cl max for root and tip = (1.744, 1.699) which has estimated by using the Xfoil-data[12] and it looks different due to fast estimation by designers.



FIGURE 6. The effect of Mach number on the 2D lift curve slope.

The Results of Lift Curve Slope 3D Prediction

The 3D lift curve parameters have calculated according to the Gudmundsson [3], DATCOM[4], Raymer [12] and Roskam [15] by considering the wing geometry of Cessna 172 as the model. This stage is relevant to the previous stage of 2D lift curve parameters. As can be seen in Table 3 the magnitude of lift at zero angle of attack has estimated by using ($\alpha_0 \ge CL_0$) according to the Gudmundsson [3].

3D Lift curve parameters	Roskam]	Raymer And Gudmundsson	Helmbold	DATCOM
Lift curve slope $C_{L\alpha}$	4.885	4.899	4.828	
Maximum lift coefficient $C_{L max}$	1.69	1.543 1.520		1.522
Angle of attack for $C_{L max}$		18.62		18.62
Zero-lift angle of attack α_0	-2	-2		
Lift at zero angle of attack CL ₀	0.1704	0.171		
Stall angle of attack for wing		16		17.41

TABLE 3. The 3D lift curve slope parameters estimation

As can be shown in Table 4 the lift curve slope has calculated by using different Oswald efficiency and velocity that obtained from Grosu [18], E. Obert [19] as well. According to these results in the Figure 7, we see the values of Helmbold, John G and lifting line theory are approached to each other at the stall speed of aircraft and aspect ratio of Cessna 172S.

TABLE 4. The comparison of methods for estimation 3D lift curve slope.							
velocity	E. Obert	Grosu C _{La}	Polhamus $C_{L\alpha}$	Helmbold	John G		
25.48128	4.714	4.817	3.48567041	4.828	4.829		
30	4.714	4.792	3.48565468	4.828	4.833		
35	4.714	4.749	3.48563418	4.828	4.838		
40	4.714	4.684	3.48561039	4.828	4.844		
45	4.714	4.592	3.48558323	4.828	4.850		
50	4.714	4.469	3.48555265	4.828	4.858		
55	4.714	4.31468402	3.48551855	4.828	4.866		
60	4.714	4.12660544	3.48548085	4.828	4.875		
64.374	4.714	3.93682263	3.48544482	4.828	4.883		
69.374	4.714	3.69520089	3.48540007	4.828	4.894		
74.374	4.714	3.43362084	3.48535137	4.828	4.906		



FIGURE 7. The comparison of methods between lift curve slope of the wing and different aspect ratio.



FIGURE 8. The effect of velocity on the max lift coefficient based on Raymer.

Comparison of 2D with 3D Lift Curve Slope

The 2D and 3D lift curve slope has estimated in previous sections where the primary purpose of this comparison is to get the maximum lift coefficient, zero angle of attack, stall angle of attack. According to the Helmbold, Roskam [15], Raymer[11] and Polhamus. In short, these results are considering new methods to get lift curve parameters depending on the stall speed and maximum speed of Cessna 172S, and Mach number as well. The results of 2D lift curve parameters have estimated by using Xfoil-data[12] or Von Doenhoff and Albert Edward [10], whereas the 3D lift curve parameters have estimated by using Equation [5, 6, 7] that obtained from DATCOM[4]. As can be shown in the figures those results have predicted carefully, and they are validated for Cessna 172S based on the stall speed and max speed.



FIGURE 9. Comparison of the lift-curve slope of a two-dimensional airfoil with a finite wingspan (3D) at max speed.



FIGURE 10. Comparison of the lift-curve slope of a two-dimensional airfoil with a finite wingspan at stall speed.



FIGURE 11. Comparison of the lift-curve slope of a two-dimensional airfoil with a finite wingspan at stall speed.



FIGURE 12. Comparison of the lift-curve slope of a two-dimensional airfoil with a finite wingspan (3D) at max speed.



FIGURE 13. Comparison of the lift-curve slope of a two-dimensional airfoil with a finite wingspan (3D) at stall speed.



FIGURE 14. Comparison of the lift-curve slope of a two-dimensional airfoil with a finite wingspan (3D) at max speed.

CONCLUSION

The geometry of aircraft has been presented as Cessna 172S. The airfoil of Cessna has adjusted such as NACA 2412 to estimate 2D prediction of aerodynamic properties for instance; lift curve slope, maximum lift coefficient and zero angle of attack. Various methods have used to estimate 3D prediction of aerodynamics characteristic of Cessna 172S such as lift curve slope, maximum lift coefficient and angle of attack by using DATCOM method and multi references. The sweep angle of Cessna 172S and mean aerodynamic chord has been estimated by using the geometrical method and theoretical analysis.

REFERENCES

- Cheeseman, I., Fluid-Dynamic Drag: Practical Information on Aerodynamic Drag and Hydrodynamic Resistance. SF Hoerner. Hoerner Fluid Dynamics, Brick Town, New Jersey. 1965. 455 pp. Illustrated. \$24.20. The Aeronautical Journal, 1976. 80(788): p. 371-371.
- Prandtl, L., *Tragflügeltheorie. I. Mitteilung.* Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, 1918. p. 451-477.
- 3. Gudmundsson, S., General aviation aircraft design: Applied Methods and Procedures. 2013: Butterworth-Heinemann.
- 4. Williams, J.E. and S.R. Vukelich, *The USAF stability and control digital dATCOM. Volume I. Users manual.* 1979, DTIC Document.
- 5. Etkin, B. and L. Reid, *Mean Aerodynamic Chord, Mean Aerodynamic Center, and ac m C.* Dynamics of Flight: Stability and Control, 3rd ed., Wiley, New York, NY, 1996: p. 357-363.
- 6. Etkin, B. and L.D. Reid, *Dynamics of flight: stability and control*. Vol. 3. 1996: Wiley New York.
- 7. Anderson, J.D., Aircraft performance and design. 1999: WCB/McGraw-Hill.
- 8. Helmbold, H., Der unverwundene ellipsenflugel als tragende flanche. Jahrbuch, 1942: p. I111-I113.

- 9. Lowry, J.G. and E.C. Polhamus, A method for predicting lift increments due to flap deflection at low angles of attack in incompressible flow. 1957.
- 10. Abbott, I.H. and A.E. Von Doenhoff, *Theory of wing sections, including a summary of airfoil data.* 1959: Courier Corporation.
- 11. Raymer, D.P., Aircraft Design: A Conceptual Approach. 2012: American Institute of Aeronautics and Astronautics.
- 12. DRELA, Xfoil
- 13. Jenkinson, L.R., P. Simpkin, and D. Rhodes, *Civil jet aircraft design*. 1999: American Insitute of Aeronautics and Astronautics.
- 14. Young, A.D., *The Aerodynamic Characteristics of Flaps*. 1947, AERONAUTICAL RESEARCH COUNCIL LONDON (ENGLAND).
- 15. Roskam, J. and C.-T.E. Lan, Airplane aerodynamics and performance. 1997: DARcorporation.
- 16. reports, N. Bertin, J.J. and R.M. Cummings, Aerodynamics for Engineers. 2013: Pearson.
- 17. GROSU, CALCULUL SI CONSTRUCTIA AVIONULUI I. GROSU 2 VOLUME.
- 18. Obert, E., Aerodynamic design of transport aircraft. 2009: IOS press.
- 19. Houghton, E.L. and Carpenter, P.W., 2003. Aerodynamics for engineering students. Elsevier.
- Chernyshev, Sergey L., Sergey V. Lyapunov, and Andrey V. Wolkov. "Modern problems of aircraft aerodynamics." *Advances in Aerodynamics* 1, no. 1 (2019): 1-15.
- 21. Sóbester, A. and Forrester, A.I., 2014. Aircraft aerodynamic design: geometry and optimization. John Wiley & Sons.
- 22. Küchemann, D., 2012. The aerodynamic design of aircraft. American Institute of Aeronautics and Astronautics,
- 23. Nicolai, L.M. and Carichner, G.E., 2010. Fundamentals of aircraft and airship design: Volume I-aircraft design. American Institute of Aeronautics and Astronautics, Inc..
- Min, S., Harrison, E., Jimenez, H. and Mavris, D.N., 2015. Development of aerodynamic modeling and calibration methods for general aviation aircraft performance analysis-a survey and comparison of models. In 15th AIAA Aviation Technology, Integration, and Operations Conference (p. 2853).