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Optimal control for a vibration control system with dead zone compensate

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ABSTRACT

Purpose: This paper aims to improve an active suspension system of vehicles by developing an optimal control strategy.

Design/methodology/approach: This work proposes a Linear Matrix Inequality (LMI) hybrid based on Liner Quadratic Integral LQI. The LMI-LQI hybrid closed-loop control is used to enhance the main parameters for the closed-loop control of the active suspension system to compensate for the dead zone nonlinearity effect in the actuator and enhance the dynamic performance of the system. An active suspension system of a quarter-vehicle with 3 DOF is considered to examine the system.

Findings: MATLAB/Environment was used to simulate a comparison between the proposed active control LMI-LQI system with dead zone input performance and active control LMI-LQI system performance with passive system performance.

Research limitations/implications: It is concluded that the proposed hybrid control improves the system performance in terms of ride comfort and safety by reducing the RMS (root mean square) seat acceleration by 93% for the LMI-LQI control system with dead zone input and 97% for the LMI-LQI system compared to the passive system. In addition, the suspension travel is reduced by 82% compared to the passive system.

Originality/value: The LMI-LQI control technique is proposed to design active suspension systems. According to the simulated results, the controller action is robust and realisable because it has the potential to minimise the nonlinear effect of the dead zone and the vertical acceleration, thus enhancing ride comfort.

Keywords: Active suspension, Dead zone nonlinearity, LMI, Quarter-vehicle model, LQI

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ANALYSIS AND MODELLING

1. Introduction

Recently, there has been great attention, in both academic and industrial fields, on advanced suspension systems development, considered an essential part of vehicles, to obtain a comfortable and safe ride. The suspension system eliminates the vibration transmitted to the vehicle body and caused by road conditions. There is research interest in developing suspension systems, divided into passive, active, and semi-active suspension systems, especially the active system, distinguished by its ability to modify energy storage by employing an additional component. The actuator is in conjunction with the suspension elements (the spring and damper) to add or dissipate control force from the system [1].

In active suspension design, suspension element models are frequently non-linear, which makes the design challenging to develop and uncertainties [2]; furthermore, according to literature, the proposed control strategies for developing active system, such as optimal control [3], sliding mode control [4,5], adaptive control [6], and another technique in [7,8], the actuator is assumed to be ideal, the functional actuator is not ideal, dead-zone, hysteresis, and saturation frequently affect them [9].

The performance of control systems can be greatly undermined by dead-zone characteristics often unknown. Industrial hydraulic actuators and electric motors have complex dead zone nonlinearity. Although dead-zone input is highly prevalent, it is important not to ignore it [10].

In many works of literature, the researchers proposed a control system method to compensate for the dead zone and hysteresis effect on actuators in various systems, such as the Inverse dead-zone model [11], adaptive control [12], backstepping control [13], and their combinations. In [14], the researchers propose an adaptive feedback controller for the symmetric dead-zone input problem by modelling the dead-zone input as a combination of linear and disturbance-like terms. Also, in [15], the researchers create a state feedback controller to solve the issue of non-symmetric dead-zone input.

As mentioned above, to enhance and improve raid comfortable and road holding, the active suspension exceeds the passive and sime-active systems. Due to their nonlinear and complex character, mathematical models for active suspension systems are notoriously difficult to establish.

This paper proposed a robust control method to enhance the ride comfortable and road holding of a vehicle by using Linear-Matrix-Inequality (LMI) based on Liner Quadratic Integral LQI simulate by using MATLAB\Simulink environment. LQI is an optimal control technique used to calculate feedback gain, which stabilises the dynamic performance, and LMI eliminates the uncertainties in the system's parameters. For the solution of LMI equations, a form of quadratic Lyapunov function is employed. This function demonstrates the system's stability and can also be used to satisfy different performance requirements.

2. Mathematical model

A quarter-vehicle suspension system with 3DOF was used to mould an active suspension system. As shown in Figure 1, the active quarter vehicle suspension system model consists of three masses, m sub 1 for seat mass supported by passive element spring k_1 and damper c_1. The second is sprung mass m_2, which is supported by passive element spring k_2 and damper$c_2$in addition to control actuator force F_a. The last one is unsprung mass, which was the wheel, tire masses, and the tire modelling as spring k_3. The parameters X_{se} , X_s and X_{us} are the vertical displacements of the seat, sprung and unsprung masses from their static positions, while $X_r$is road excitation displacement, which considers the disturbance that affected the system.

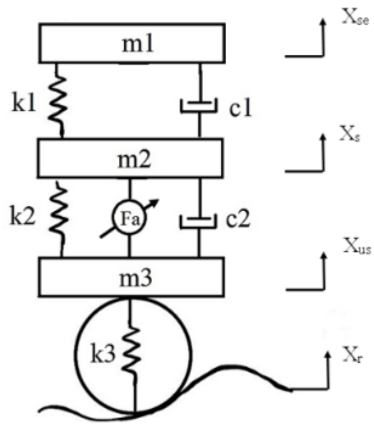


Fig. 1. Model of active suspension system for quarter vehicle

Applying Newton's 2nd law of motion to FBD in Figure 1 obtains the EOM for the active system, as shown below.

$$\ddot{x}_{se} = \frac{1}{m_1}[-c_1(\dot{x}_s - \dot{x}_{se}) - k_1(x_s - x_{se})] \quad (1)$$

$$\ddot{x}_s = \frac{1}{m_2}[-c_1(\dot{x}_s - \dot{x}_{se}) - c_2(\dot{x}_s - \dot{x}_{us}) - k_1(x_s - x_{se}) - k_2(x_s - x_{us}) - F_a] \quad (2)$$

$$\ddot{x}_{us} = \frac{1}{m_3}[-c_2(\dot{x}_{us} - \dot{x}_s) - k_2(x_{us} - x_s) - k_3(x_{us} - x_r) + F_a] \quad (3)$$

3. Control design

3.1. State space representation

Six output variables are defined:

$x_1 = x_{se}, x_2 = \dot{x}_{se}, x_3 = x_s, x_4 = \dot{x}_s, x_5 = x_{us}, x_6 = \dot{x}_{us},$

Two input variables are defined:

$$u_1 = F_a \;\; and \;\; u_2 = x_r$$
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \tag{4}$$

$$A = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{k_1}{m_1} & \frac{c_1}{m_1} & -\frac{k_1}{m_1} & -\frac{c_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_1}{m_2} & \frac{c_1}{m_2} & \frac{-k_1-k_2}{m_2} & \frac{-c_1-c_2}{m_2} & \frac{k_2}{m_2} & \frac{c_2}{m_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{k_2}{m_3} & \frac{c_2}{m_3} & \frac{-k_2-k_3}{m_3} & -\frac{c_2}{m_3} \end{vmatrix} \quad B = \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{-1}{m_2} & 0 \\ 0 & 0 \\ \frac{1}{m_3} & \frac{k_3}{m_3} \end{vmatrix}$$

Recall that the two outputs or measurements are specified as $y_1 = x_{se}$ *and* $y_2 = \ddot{x}_{se}$

$$C = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k_2}{m_3} & \frac{c_2}{m_3} & \frac{-k_2-k_3}{m_3} & -\frac{c_2}{m_3} \end{vmatrix}$$

D=[0]

where, A is the state matrix, B is the input matrix, C is the output matrix, and D is the feedforward matrix.

3.2. Mathematical description of a dead-zone

It can obtain a mathematical equation of a dead zone in terms of the Figure 2 shown below, which represents the definition of the dead zone in the actuator [16].

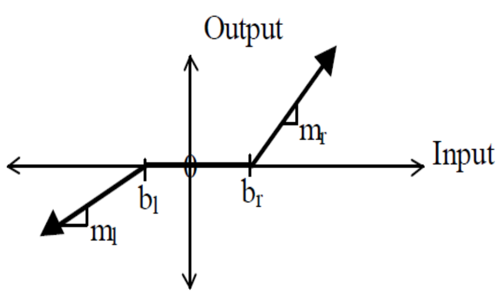


Fig. 2. A nominal definition of a dead-zone

The output, $u_{(t)}$, is related to the input, $v_{(t)}$, by the following expression, which describes the dead zone of the actuator used in the control systems

$$u_{(t)} = \begin{cases} m_r\big(v_{(t)} - b_r\big), v_{(t)} \geq b_r \\ 0, b_1 < v_{(t)} < b_r \\ m_1\big(v_{(t)} - b_1\big), v_{(t)} \leq b_1 \end{cases} \tag{5}$$

If dead zone is taken into account, the dynamic model of the active suspension system is as follows:

$$\dot{x} = Ax + B_1 \mathrm{Fa}_{\emptyset} + B_2 x_r \tag{6}$$

where $\mathrm{Fa}_{\emptyset}$ represents the output signal after the dead zone effect which is $u_{(t)}$ in Figure 2, and $\mathrm{Fa}_{\emptyset}$ is the input signal.

3.3. LQR controller

The Linear Quadratic Regulator (LQR) problem aims to design optimal controllers for linear systems, which is described by the following relationship [17]:

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t) \tag{7}$$

$$y(t) = Cx(t) \tag{8}$$

where *x(t)* is the state variable vector, *u(t)* is the input, *y(t)* is the output vector, and *d(t)* is the external disturbance to the system.

The linear control law for a linear quadratic regulator is described as follows:

$$u = -k\,X \tag{9}$$

Where this linear control law that achieves:

1 – stabilization of the linear system according to Lyapunov's condition.

2 – moving the system from its initial state to its desired state while minimising the performance indicator described by the following relationship:

$$J = \int_0^{\infty} (X^T(t)QX(t) + u^T(t)Ru(t))\,dt \tag{10}$$

The quadratic performance index in relation (9) achieves the minimum error and the minimum power supplied to the system.

where Q: is the state weighting matrix that must be a square positive semi-definite matrix, R: is the control weighting matrix that must be a square positive definite matrix.

By substituting relationship 9 into relationship 10 we get:

$$J = Tr((Q + k_c' R k_c)L_c) \tag{11}$$

where ($L = \|x\|$).

Lyapunov's condition for stability is described by the following relationship

$$(A - Bk)L_c + L_c(A - Bk)' + x_i'x_i = 0 \quad (12)$$

By combining the Lyapunov condition of stability (8) with the cost function (7), we get the Lagrange function:

$$J_a = P_c\left((A - Bk)L_c + L_c(A - Bk)' + x_i'x_i\right) + (Q + k'Rk)L_c \quad (13)$$

where (P_c) is the Lagrange variable, which is a positive, semi-definite matrix.

Finding the minimum limit of the objective function according to the constraint (Lyapunov condition of stability) is equivalent to finding the minimum limit of the Lagrange function J_a, That is, the search for k that achieves making all the partial derivatives of the first order of the Lagrange function J_a with respect to the unknowns (L_c, P_c, k) equal to zero, this leads to:

$$k = (1/R)B'P_c \quad (14)$$

$$(A - Bk)'P_c + P_c(A - Bk) + k'Rk + Q = 0 \quad (15)$$

The last equation is called the Rickett algebraic equation, which leads to the calculation of, and then by substituting in 10, k is calculated.

3.4. LMI-based LQR

For linear systems with uncertainty models where system parameters may be variable or unknown but are limited within a known range or there is external disturbance [18], the LQR method for calculating feedback constants should be modified to a method of optimising constraints without LMI optimisation, as the methodology ranks as a powerful design tool that provides solutions for many of convex problems [19].

Taking into account the uncertainty, the matrices of systems A and B can be represented by A_ρ and B_ρ where:

$$A_\rho = \left(A_0 + \sum_{j=1}^{l} A_j\,\delta_j\right), \sum_{j=1}^{l} \delta_j = 1 \quad (16)$$

$$B_\rho = \left(B_0 + \sum_{i=1}^{l} B_j\,\delta_j\right), \sum_{j=1}^{l} \delta_j = 1 \quad (17)$$

A_0, B_0 are the normal state, input matrix, δ_i is the scalar of the uncertain parameter corresponding to uncertain matrices A_j and B_j.

Here, instead of searching for the value of P_c that satisfies the Rickett algebraic equation, it is searched for the value of P_c that satisfies the following inequality:

$$(A_\rho - Bk)'P_c + P_c(A_\rho - Bk) + k'Rk + Q < 0 \quad (18)$$

The above inequality must be satisfied at all vertices of the space, including the uncertainty state space of the system.

Because there are $P_c k$ and k'Rk terms in this inequality, it does not represent an LMI problem. Fortunately, changing some unknowns and other fixes can be converted into an LMI problem. assume $S_c = P^{-1}$ and replace the product (kS_c) with an N. Then, multiply both sides of the mentioned inequality from the left and right by the term S_c to get the following equation [20]:

$$\left(A_\rho S_c + BN\right) + \left(A_\rho S_c + BN\right)' + S_c Q S_c + N'RN < 0 \quad (19)$$

The above inequality can be represented as LMI using Schur form as follows:

$$\begin{bmatrix} A_\rho S_c + S_c A_\rho' + BN + N'B & N' & S_c \\ N & -R^{-1} & 0 \\ S_c & 0 & -Q^{-1} \end{bmatrix} < 0 \quad (20)$$

Returning to the dynamical model of the system described in relation to an ear, the system can be modelled as follows:

$$\dot{x} = A_\rho x + B_\rho \mathrm{Fa} + B_2 x_r \quad (21)$$

Thus, depending on the MATLAB programme using the LMI code, it is possible to obtain the control gain that achieves good performance and stability of the system despite the uncertainty resulting from the dead zone and the uncertainty resulting from the change of system parameters.

To improve the dynamic performance of the system, an integral part can be added to the control law by expanding the state space of the system and adding a new state variable given as follows:

$$\zeta = \int e\,dt \quad (22)$$

So, the new state space of the system becomes as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\zeta}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \zeta(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \quad (23)$$

The LMI-based LQI controller block diagram illustrations in Figure 3.

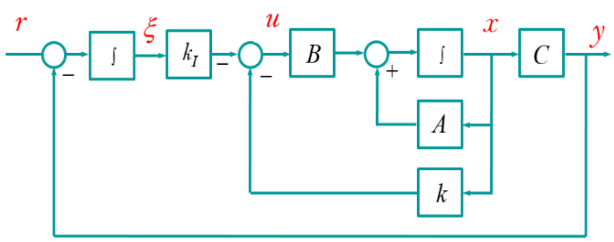


Fig. 3. The block diagram of the LMI-based LQI controller system

4. Simulation results

When the active system was subjected to road input, the proposed controller technique's efficiency, reliability, and potential to improve system performance were evaluated. Consequently, two road types were utilised for road profile disturbance, as shown in Figure 4. The first one is the pump profile [21]. The road equation is:

$$x_r(t) = \begin{Bmatrix} (r(1-\frac{\cos 8\pi t}{2}) & 0.25 \le t \le 0.5 \\ 0 & otherwais \end{Bmatrix} \tag{24}$$

(r) = 8 cm (bump height).

The other one is the IOS class C random profile [22]. The road equation is:

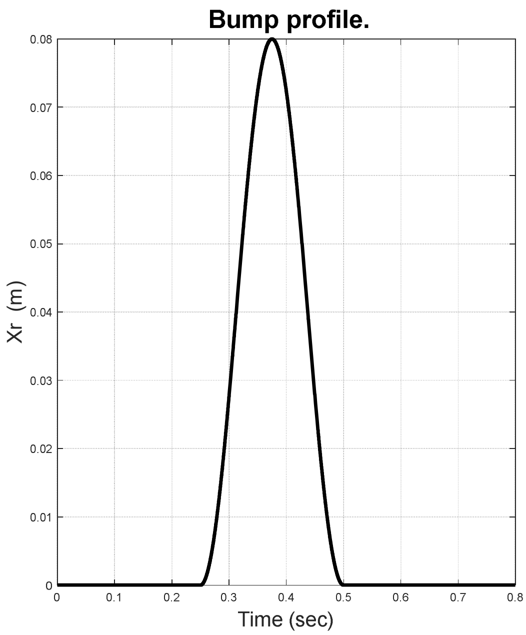
$$x_r(t) = \sum_{n=1}^{N} A_n \sin(n w_0 t - \vartheta_n) \tag{25}$$

According to ISO 2631 [23], ride comfort and safety performance are measured. The RMS values of the seat and sprung mass acceleration are used to analyse how comfortable the ride is. To validate the effectiveness of the LMI-LQI controller design, simulation studies were carried out in the MATLAB/Simulink environments. The parameters used in the simulation are shown in Table 1 [24].

Table 1.
Quarter vehicle suspension element parameters

Parameter	Value	Parameter	Value
m_1	70 Kg	k_2	16000 N\m
m_2	345 Kg	c_1	800 N.s\m
m_3	40.5	c_2	1500 N.s\m
k_1	8000 N\m	k_3	190000 N\m

Figures (5-7) depict the dynamic response of a quarter vehicle suspension system, with comparisons of the passive response, LMI-LQI active control response, and LMI-LQI active control response under the action of dead zone input effect response. It can be seen from Figure 5, which represents the seat displacement response and sprung displacement in cases of bumpy and random road profiles, respectively, that the displacement was significantly reduced by LMI-LQI active control and LMI-LQI active control with dead zone input compared with the passive system according to the damping force. In terms of pump profile and random profile, the amount of the reduction is 85%. Additionally, Despite the variance that appeared in the random profile response, it is also possible to see that the responses of the two active controllers, the LMI-LQI controller and the LMI-LQI controller with dead zone input, are nearly identical. It shows that the controller was successful in eliminating the effect of the actuator's dead zone effect.



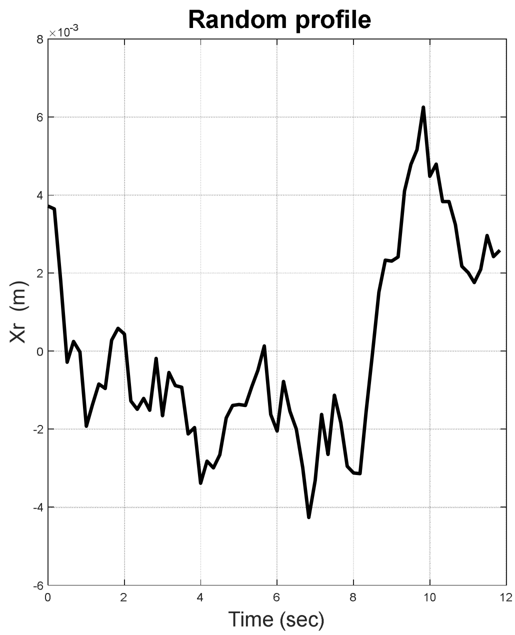


Fig. 4. Road profile

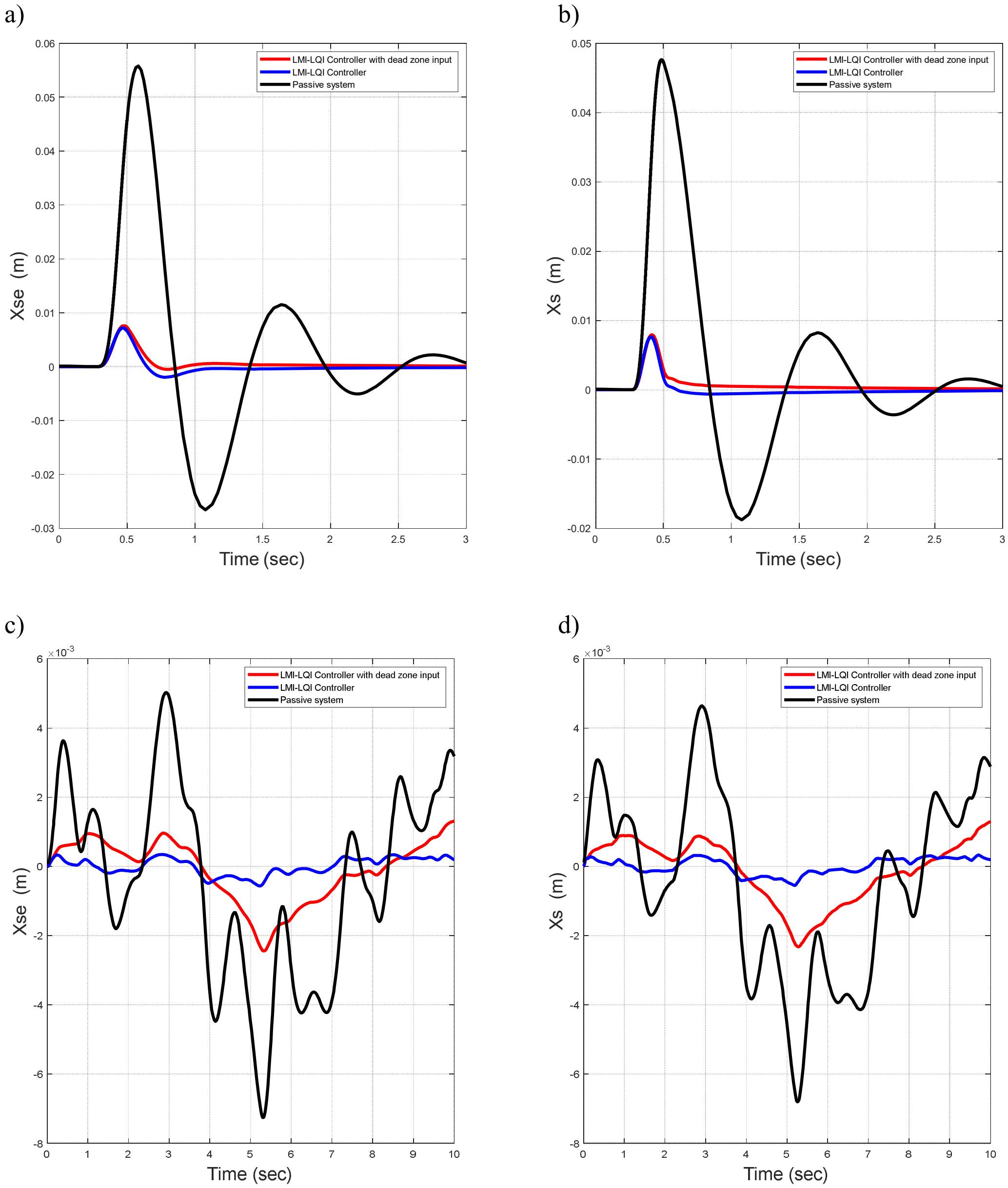


Fig. 5. Displacement response of: a) seat in case of bump road, b) sprung in case of bump road, c) seat in case of random road, d) sprung in case of random road

Figure 6 below show the seat acceleration response and sprung acceleration in bumpy and random road profiles, respectively. It is possible to see a reduction in acceleration response relative to a controller action under the effect of the LMI-LQI active control and LMI-LQI active control with dead zone input compared with the passive system. The acceleration reduction is approximately 73% for the pump profile and 80% for the random profile. Also, the responses of the two active controllers, the LMI-LQI controller and the LMI-LQI controller with dead zone input, are almost the same, and that achieves more comfortable riding.

Since the road holding is associated with suspension travel, it can see in Figure 7 that the suspension travel has a reduction by 82% in both the LMI-LQI controller and the LMI-LQI controller with dead zone input systems. That chevied safer for vehicle ride.

Also, here it can analyse and characterise the effectiveness of the quarter vehicle suspension system in

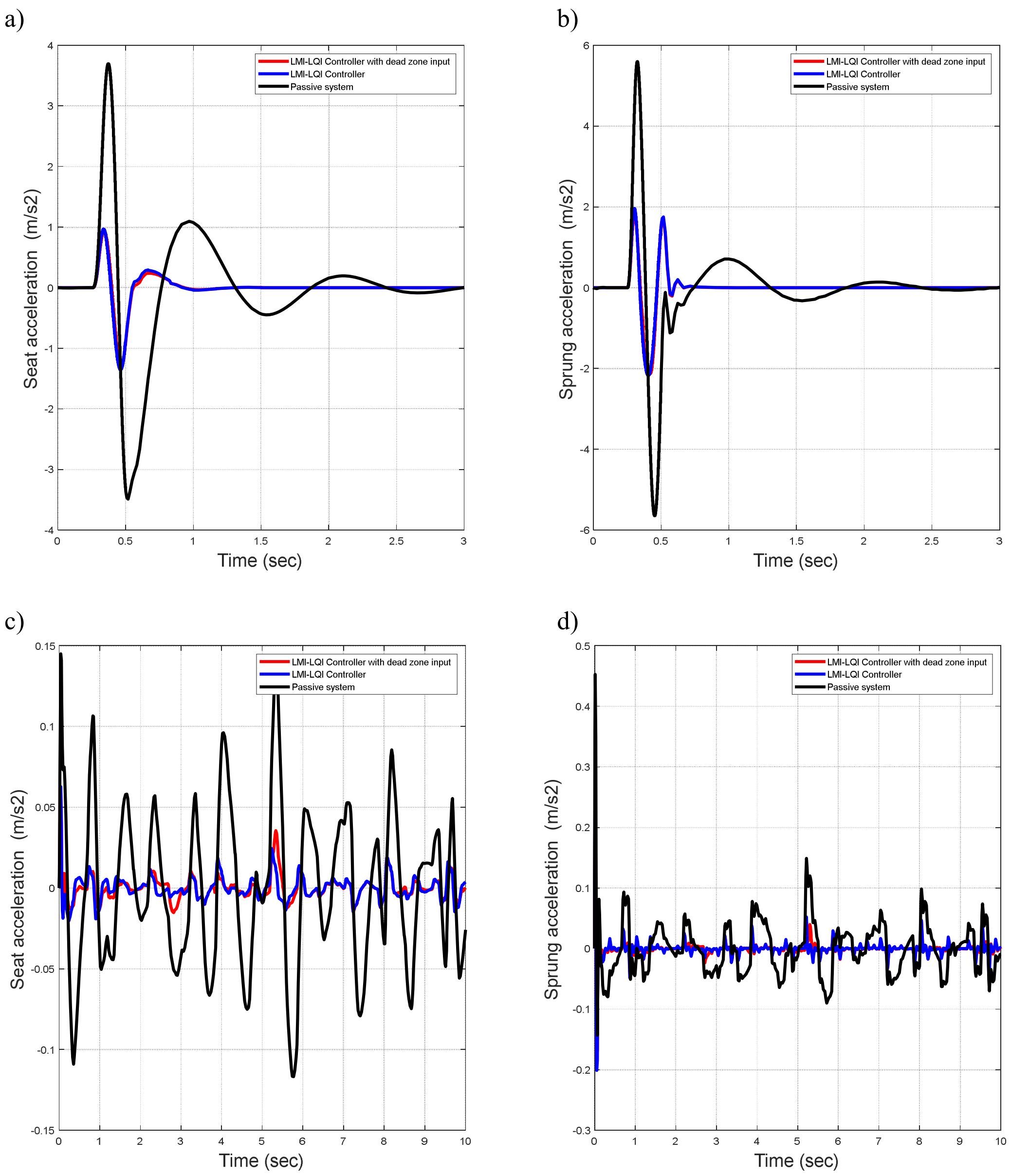


Fig. 6. Acceleration response of: a) seat in case of bump road, b) sprung in case of bump road, c) seat in case of random road, d) sprung in case of random road

satisfying ISO 2631-1 performance standards for vibration reduction while also enhancing ride quality. So, the RMS acceleration of LMI-LQI controller and the LMI-LQI controller with dead zone input are determined to be 0.000066 m/s^2 and 0.000062 m/s^2, respectively, while for passive is 0.0025 m/s^2 in pump road and 0.00037 m/s^2 and 0.001 m/s^2 respectively while for passive is 0.025 m/s^2 in random road. From comparing those values with ISO 2631-1 performance standards in [23], it can say that the suggested control has RMS acceleration lower than the passive system. It can see that in the case of using the pump profile, the RMS acceleration was reduced by 97% for LMI-LQI controller system and by 96% for the LMI-LQI controller system with dead-zone input. Furthermore, in the case of using a random profile, the RMS acceleration was reduced by 97% for LMI-LQI controller system and by 93% for the LMI-LQI controller system with dead zone input.

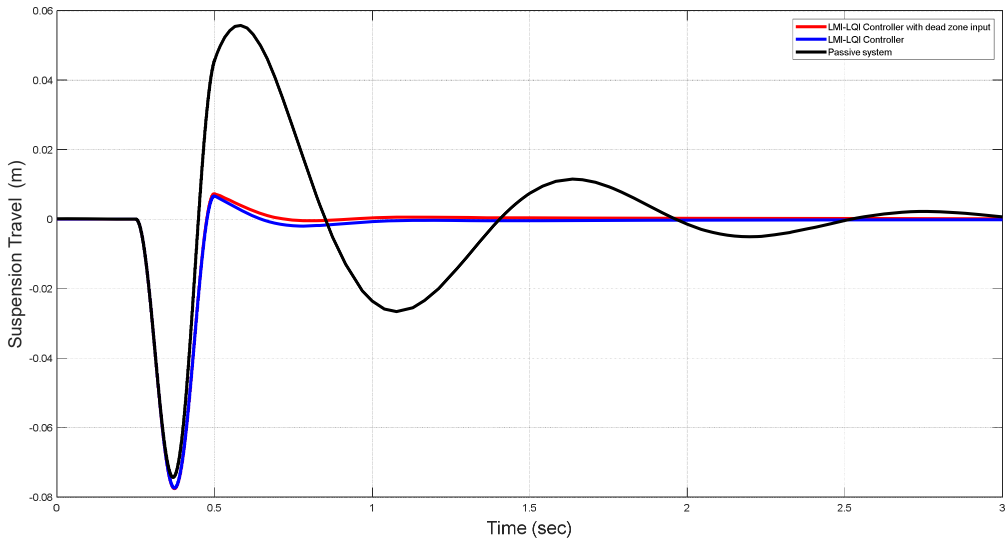


Fig. 7. Suspension travel response

In summary, the proposed active control of LMI-LQI successfully reduced quarter vehicle displacement and acceleration, eliminated signal oscillations, and reached a steady state more instantly; it also eliminated the dead zone nonlinearity effect and achieved a comfortable and safe ride.

5. Conclusions

Developing a new control technique is to satisfy a wide range of customer requirements, such as improved comfort, increased reliability, and safety. A lot of time and energy has gone into developing control systems to achieve that. It has been proposed to use the LQI controller technique and the LMI technique to control suspension systems. The LQI control model for the 3 DOF model of a quarter vehicle, which can consider parameter uncertainty such as the nonlinearity effect in the actuator, is introduced. In contrast, the LMI technique can ensure the system's stability and eliminate the nonlinearity effect.

Compared to the passive suspension system, the hybrid LMI-LQI active control simulation result achieved stability and faster response in terms of displacement and acceleration and eliminated the dead zone effect. It led to a more comfortable ride and improved safety by reducing the RMS (root mean square) seat acceleration by 93% for the LMI-LQI control system with dead zone input and 97% for the LMI-LQI system compared to the passive system. In addition, the suspension travel is reduced by 82% compared to the passive system.

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