The modified trapezoid approach is based on both harmonic and arithmetic means

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ABSTRACT

This paper's goal is to present a revised guideline for data point numerical integration. This approach is based on computations of the harmonic and arithmetic means of the points that make up our interval to evaluate the precision of our approach. Numerous comparisons are computed. This procedure produces highly accurate results.

Keywords: Harmonic mean, Heron Ian mean, trapezoidal rule, arithmetic mean, and numerical integration

1. Introduction.

The core notion underlies the majority of numerical integration techniques. That is, the definite integral occurs if the function g(x) in the interval [c,f] is continuous and non-negative. [3]

$$\int_c^f g(x)dx \qquad (1)$$

Is the region where x = f, x = c, and k = g(x) meets the curve.

A numerical integration is any approximate solution of this magnitude. and the numerical methods used to calculate these approximations vary depending on how the area was approximated. Engineering and physics

Both require numerical integration. [1] and when scientific issues cannot be resolved by analytical means. In order to assess numerical integration , use the general quadrature rule. by whom given [3]

$$\int_c^f g(x)dx \approx \sum_{i=0}^{n} wi\, g(xi) \qquad (2)$$

By designating (n+1) intermediate points in such a way that The step size used to divide the finite interval [c,f] into subintervals is h=(f-c)/n.[3]

$$c = x0 < x1 < x2 < x3 < \cdots < xn = f$$

$$xi = x0 + ih \text{ and } (n+1)w0, w1, w2 \ldots\ldots\ldots wn$$

Consider the values for $wi = 0,1,2,3, \ldots\ldots\ldots n$

Numerical integration is frequently performed using the closed Newton-Cotes quadrature formula. The trapezoidal rule is obtained when we insert 1 for n in Newton Cotes' formula [2].

$$\int_c^f g(x)dx = \frac{h}{2}[k0 + 2(k1 + k2 + k3 + \cdots + kn - 1) + kn] \qquad (3)$$

The Heronian mean and the arithmetic mean are the main components of the second methodology used in this study [4]. When results were compared to those obtained using the trapezoid equation, it was found that the new method yielded more accurate conclusions with lower rates of errors [4].

$$\int_c^f g(x)dx = h\,[\frac{(k0+k1)}{2} + \frac{(k1+k2)}{2} + \frac{(k2+k3+\sqrt{k2.k3})}{3} + \frac{(k3+k4+\sqrt{k3.k4})}{3} + \ldots] \qquad (4)$$

In this research , A technique is provided that was chosen after evaluating the results using the methods such as the arithmetic mean and the trapezoid , and the harmonic mean while using the arithmetic mean and mean.

contrasting the outcomes
The outcome was more precise and had a lower error percentage than the older methods [7],[9].

$$M = \int_c^f G(x)dx = h[\sum \frac{x_0+x_1}{2} + \frac{2(x_1 * x_2)}{x_2+x_1} + \frac{x_2+x_3)}{2} + \cdots] \quad (5)$$

Numerical integration is crucial for solving numerous mathematical models. In the area of applied mathematics, it has numerous uses. especially in physics, chemistry, and engineering [17], [18]. Additionally, integral computations that are challenging to do analytically are used.

2. The heron means

if $\boldsymbol{a = \{a_1, a_2, a_3, \ldots \ldots a_n\}}$

is a set of real positive numbers. Equation [10] can be broadly characterized as follows:

$$\mathrm{Her}(2, \{a, b\}) = \frac{a+b+\sqrt{ab}}{3} = (\sqrt{aa} + \sqrt{bb} + \sqrt{ab})/3 \quad (6)$$

3. Arithmetic mean: this measure is the most crucial and extensively employed, and it may be derived for the following types of data. The arithmetic mean of the ungrouped data is as follows. The mathematical mean is commonly as the product of the values divided by the total amount of values. When value n is present, they are displayed by the symbol $K_1, K_2, \ldots, K_n$, the following equation [10] is used to determine the K-symbol, which stands for the average of these values, in arithmetic form:

$$\boldsymbol{\overline{K} = \frac{K_1+K_2+\cdots+K_n}{n} = \frac{\sum_{i=1}^{n} K_i}{n}} \quad (7)$$

4. Harmonic Mean (HM):

The inverse of the average of the inverses of the values of the data. Based on all of the observations, the harmonic mean correctly balances the values by giving greater weight to the little values and less weight to the large values. On the whole,

$$\boldsymbol{HM = \left(\frac{\sum_{i-1}^{n} K_i}{n}\right)^{-1}} \quad \textbf{(8)}$$

5. modified trapezoidal rule based on harmonic and arithmetic means

For the computation of definite integrals, a modified trapezoidal rule is derived in this section [7].

$\int_C^f g(x)dx$ Over [c, d]. beyond [c, d]. The function is initially tabulated by using the step size $h = \frac{f-c}{n}$

5.1: propositions.

Let:

Z	**z0**	$\boldsymbol{z1}$	$\boldsymbol{z2}$	**………..**	$\boldsymbol{zn-1}$	$\boldsymbol{Zn}$
k	$k0$	$k1$	$k2$	………..	$kn-1$	kn

Next, calculate the harmonic and arithmetic means two times.

$$M1 = \int_{z0}^{z1} \mathrm{g(x)dx} = \mathrm{h}[\ \frac{z_0+z_1}{2}]$$

$$M2 = \int_{X1}^{z2} g(x)dx = h\,[\frac{2(z_1 * z_2)}{z_2+z_1}]$$

$$M3 = \int_{z2}^{z3} g(x)dx = h\left[\frac{z_2+z_3}{2}\right]$$

$$M4 = \int_{z3}^{z4} g(x)dx = h\,[\frac{2(z_3 * z_4)}{z_4+z_3}]$$

$$M5 = \int_{z4}^{z5} g(x)dx = h[\frac{z_4+z_5}{2}]$$

$$M6 = \int_{z5}^{z6} g(x)dx = h\ [\frac{2(z_5 * z_6)}{z_6 + z_5}]$$

Repeat the process outlined above, then include the findings.

$$M=\int_c^f g(x) = h[\frac{z_0+z_1}{2} + \frac{2(z_1 * z_2)}{z_2+z_1} + \frac{z_2+z_3}{2} + \frac{2(z_3 * z_4)}{z_4+z_3} + \cdots] \qquad (9)$$

6. Numeral Illustrations

The outcomes of the new technique, the arithmetic mean approach, and the old trapezoid rule will all be compared in this section. We are familiar with [5],[3].

Error = I precise value - approximate amount I

6.1: Example

Calculate $\int_0^1 \frac{1}{1+z^2} dx$ and compare the answers.

Exact amount of $\int_0^1 \frac{1}{1+z^2} dx = 0.6931471806$, $h = 1/4$

Z	**0**	**1/4**	**1/2**	**3/4**	1
K	1	0.9412	0.8000	0.6400	0.5000

result from the suggested method

$$M = \frac{1}{4}[0.9706 + 0.864874799 + 0.72 + 0.5614035088]$$

M = 0.779219577

Error=I 0.7854 - 0.7792 I=0.0062

Examples (1)

Examples of the Suggested Method in Comparison to the Arithmetic Mean Method for Trapezoidal Rules

Function	**Actual Value**	**Rule of the trapezoid**	**Error**	**Arithmetic Mean Technique**	**Error**	**Proposed Approach**	**Error**
$\int_1^2 \frac{1}{1+x} dx$	0.405465	0.405810	0.000425	0.40585	0.000368	0.4056624953	0.0001974953
$\int_1^2 e^x\, dx$	4.670774	4.686327	0.0155530	4.683004	0.012230	4.668063698	0.002710302
$\int_0^1 \frac{\ln(1+x)}{1+x} dx$	0.240226507	0.268530	0.028303493	0.268443	0.028216493	0.2344113189	0.0058151881
$\int_{-1}^1 \frac{dx}{\sqrt{4-x2}} dx$	1.0471975511965	1.055072914	0.0078536	1.054920249	0.00722697412	1.048995573	0.001798021804

$\int_{0}^{2} \sqrt{(4-x2)dx}$	3.141592654	3.445562556	0.303969902	3.748964181	0.607371527	2.965729243	0.175863411
$\int_{1.0}^{1.8} \frac{e^{x}+e^{-x}}{2} dx$	1.766973094	1.772859082	0.00588988	1.771391053	0.00441795908	1.771122067	0.00414897 3

7. Conclusions:

Based on the solution to several examples (1) involving exact integration and comparison of the output of our hypothetical method with the arithmetic mean method and the trapezoidal rule, it is determined that our method has the least number of errors, followed by those of the trapezoidal rule and similarly comparing the results of our hypothetical method with the mean method Arithmetic. We find that our method is less wrong than the arithmetic mean method.

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